Bayesian Calibration of the Intelligent Driver Model

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Introduction

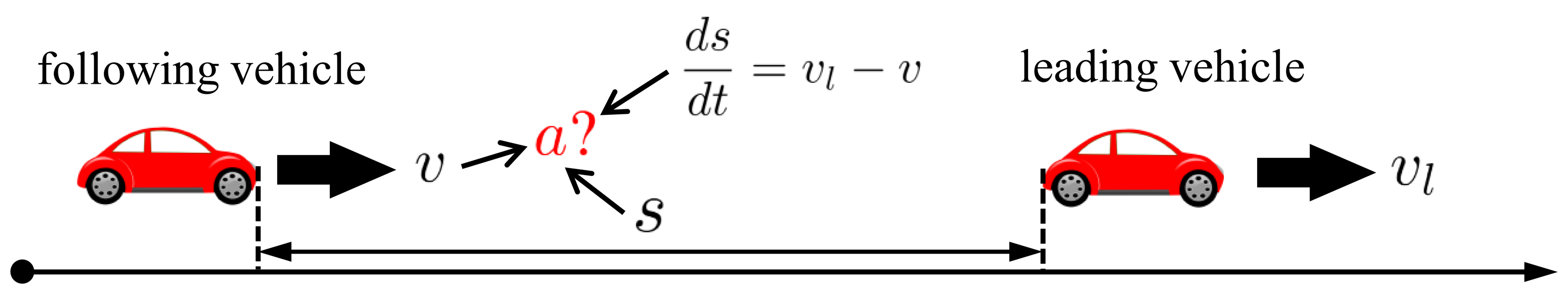

Fig. 1: Physical settings of a car-following scenario.

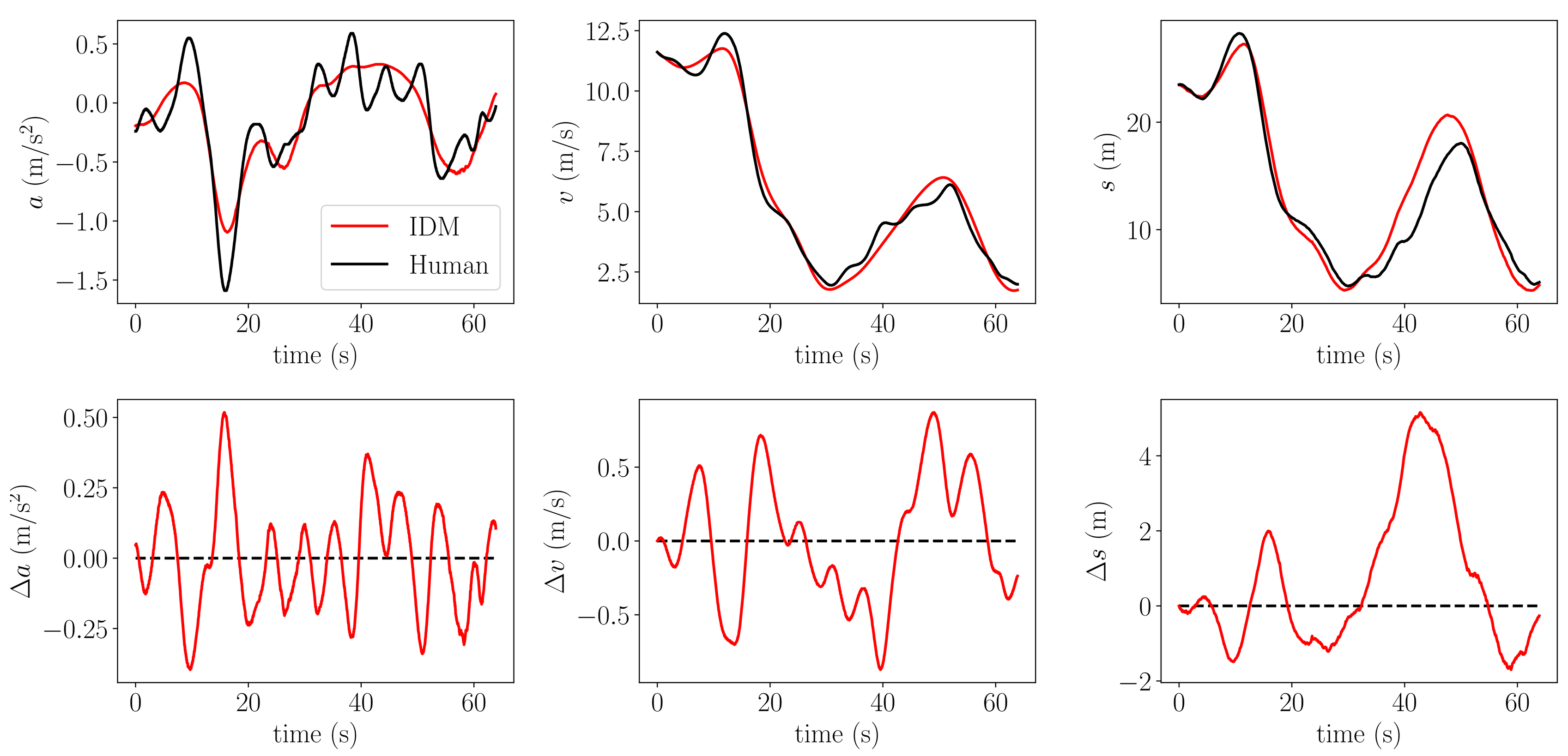

Fig. 2: The residual trends of the calibrated IDM have strong serial correlations.

The Intelligent Driver Model (IDM) is formulated as

$$a_{\text{IDM}}(v, \Delta v, s) \triangleq \alpha \left(1 - \left(\frac{v}{v_0}\right)^{\delta} - \left(\frac{s^*(v, \Delta v)}{s}\right)^2\right) \quad (1)$$

$$s^*(v, \Delta v) = s_0 + v\,T + \frac{v\,\Delta v}{2\sqrt{\alpha\,\beta}} \quad (2)$$

Calibrating the IDM $\overset{data}{\Longrightarrow}$ **Identify** $[\alpha, v_0, s_0, T, \beta]$.

☛ arXiv: **https://arxiv.org/abs/2210.03571**

☛ Code: **https://github.com/Chengyuan-Zhang/IDM_Bayesian_Calibration**

Summary

- We develop a novel Bayesian calibration approach to learn unbiased parameters and their full posterior distribution. We introduce GP to characterize the autocorrelation in residuals. This approach is applied to calibrate IDM, and shapes the form of the memory-augmented IDM (MA-IDM).
- We implement the MA-IDM with three hierarchies. Especially with a hierarchical MA-IDM, one can obtain diverse driving styles at the population level and disparate driving behaviors at the individual level by sampling from the posterior distributions of the well-calibrated hierarchical model. Therefore, we can generate enormous drivers with heterogeneous driving behaviors/styles governed by the same population distribution. Therefore, our model can help create simulations with driver/car heterogeneity.
- We introduce an unbiased stochastic simulator, which is inspired by the corresponding generative process of our Bayesian calibration approach. As a result, the simulator can produce more realistic results than those with homogeneous parameters or random parameters.

Methodology

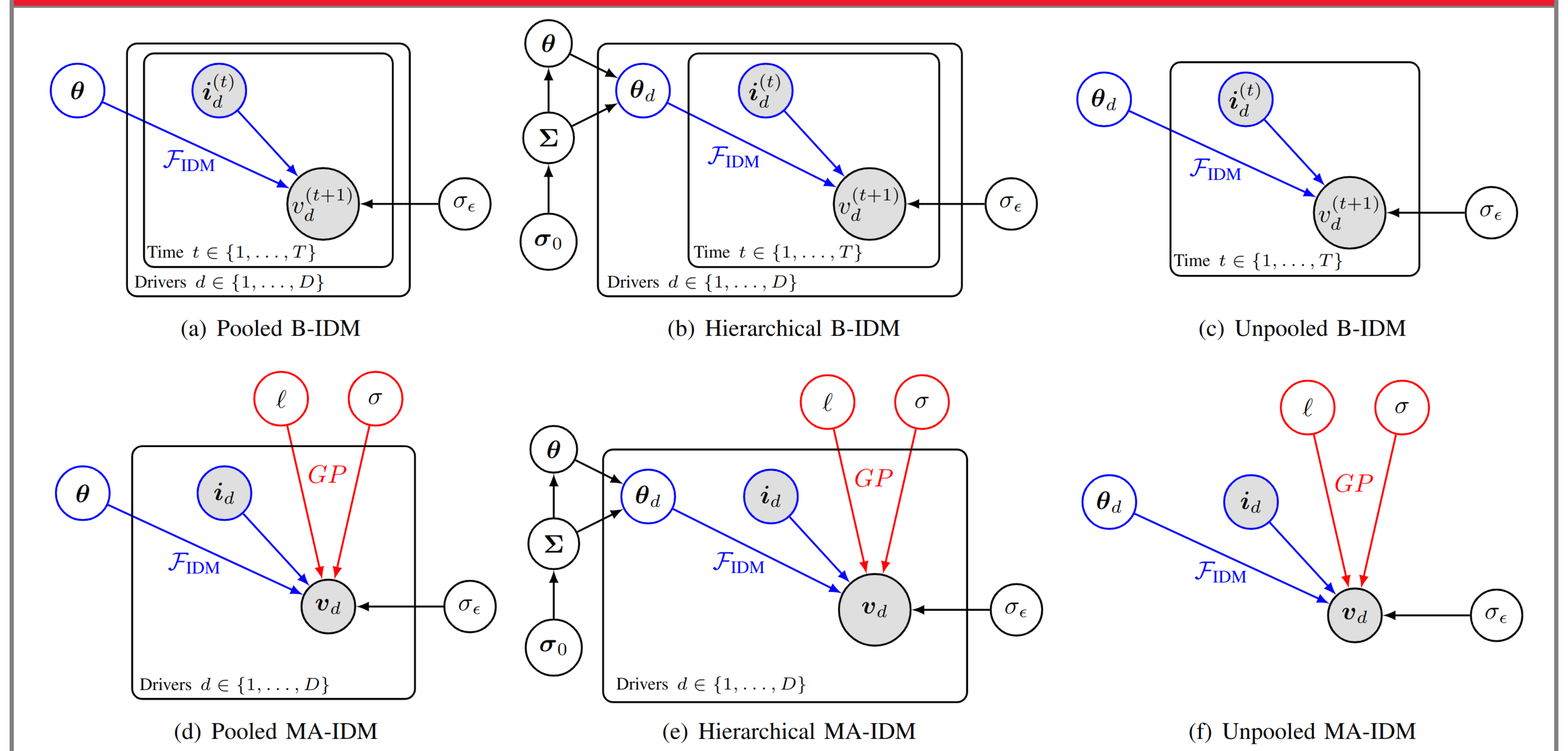

Fig. 3: Probabilistic graphical models of B-IDM and MA-IDM.

✗: $a_d^{(t)} = a_{\text{IDM}\,d}^{(t)} + \epsilon_t,\ \epsilon_t \overset{i.i.d.}{\sim} \mathcal{N}(0, \sigma_\epsilon^2)$. ✓: $a_d^{(t)} = a_{\text{IDM}\,d}^{(t)} + a_{\text{GP}\,d}^{(t)} + \epsilon,\ \epsilon_t \overset{i.i.d.}{\sim} \mathcal{N}(0, \sigma_\epsilon^2)$.

- **Main assumption:** We model the true process of the car-following actions based on three parts: (1) the mean trends $a_{\text{IDM}\,d}^{(t)}$, (2) an inertia residual $a_{\text{GP}\,d}^{(t)}$, and (3) i.i.d. observation noise ϵ. Such that we have $\boldsymbol{a}_d | \boldsymbol{i}_d, \boldsymbol{\theta}_d \sim \mathcal{N}(\boldsymbol{a}_{\text{IDM}\,d}, \boldsymbol{K} + \sigma_\epsilon^2 \boldsymbol{I})$.
- **Hierarchical Bayesian IDM (B-IDM) and MA-IDM:**

$$\begin{aligned} \boldsymbol{\sigma}_0 &\sim \text{Exp}(\lambda) \in \mathbb{R}^5 \\ \boldsymbol{\Sigma} &\sim \text{LKJCholeskyCov}(\eta, \boldsymbol{\sigma}_0) \\ \ln(\boldsymbol{\theta}) &\sim \mathcal{N}(\boldsymbol{\mu}_0, \boldsymbol{\Sigma}) \in \mathbb{R}^5 \\ \ln(\boldsymbol{\theta}_d) &\sim \mathcal{N}(\ln(\boldsymbol{\theta}), \boldsymbol{\Sigma}) \in \mathbb{R}^5 \\ \ln(\sigma_\epsilon) &\sim \mathcal{N}(\mu_\epsilon, \sigma_1) \in \mathbb{R} \\ v_d^{(t+\Delta t)} &\overset{i.i.d.}{\sim} \mathcal{N}(\mathcal{F}_{\text{IDM}}(\boldsymbol{i}_d^{(t)}; \boldsymbol{\theta}_d), (\sigma_\epsilon \Delta t)^2) \end{aligned}$$

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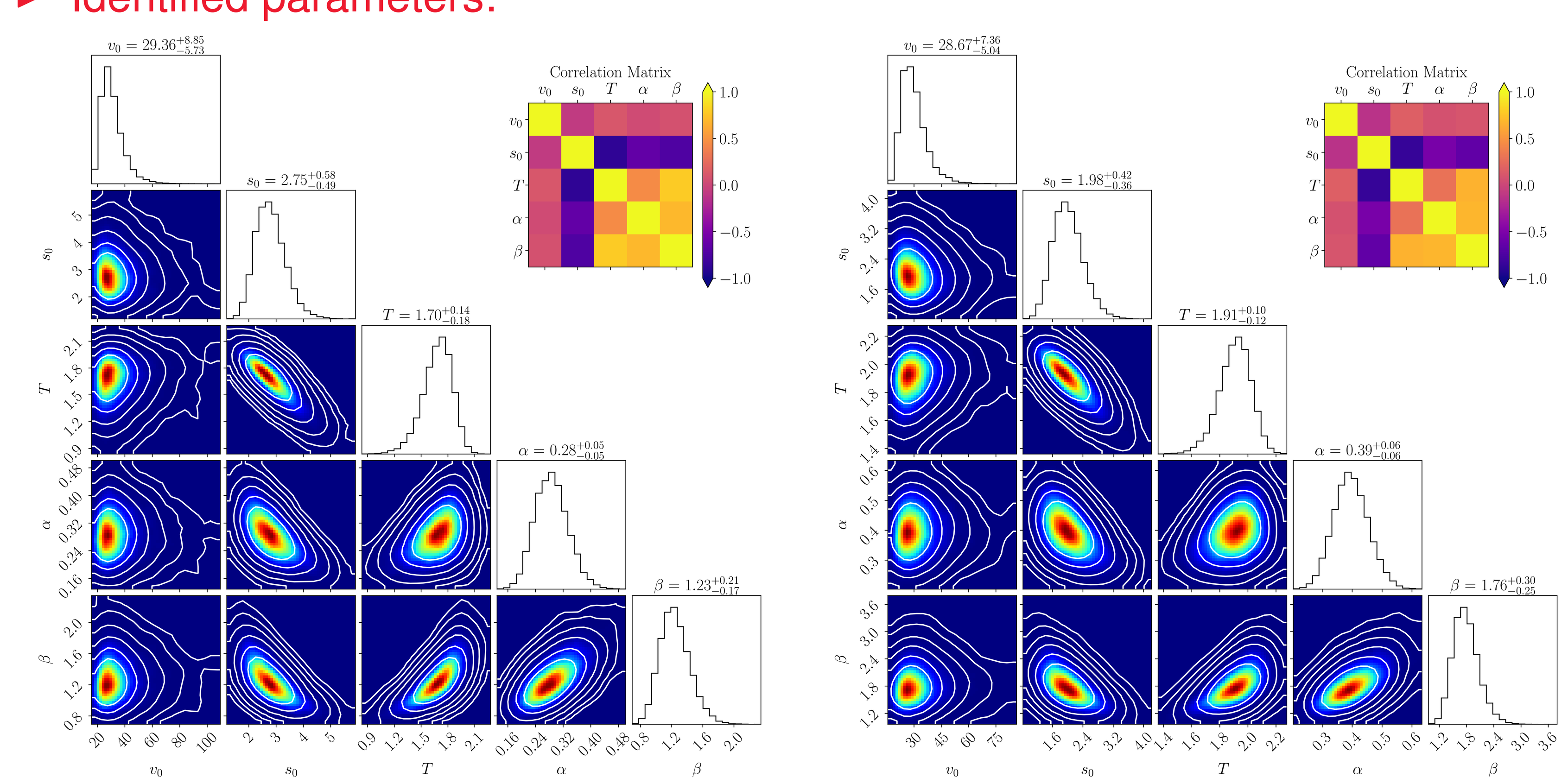
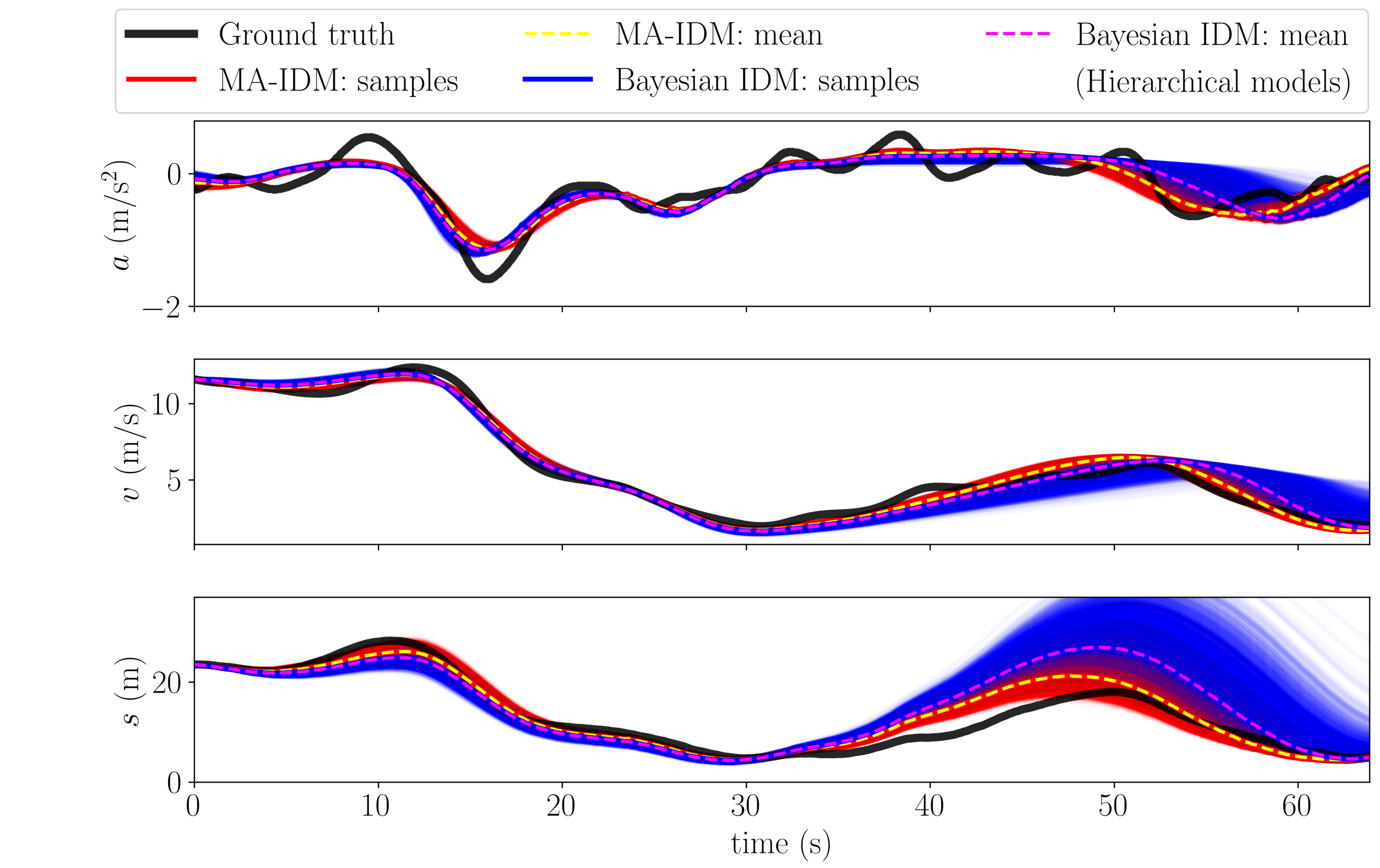
- **Identified parameters:**



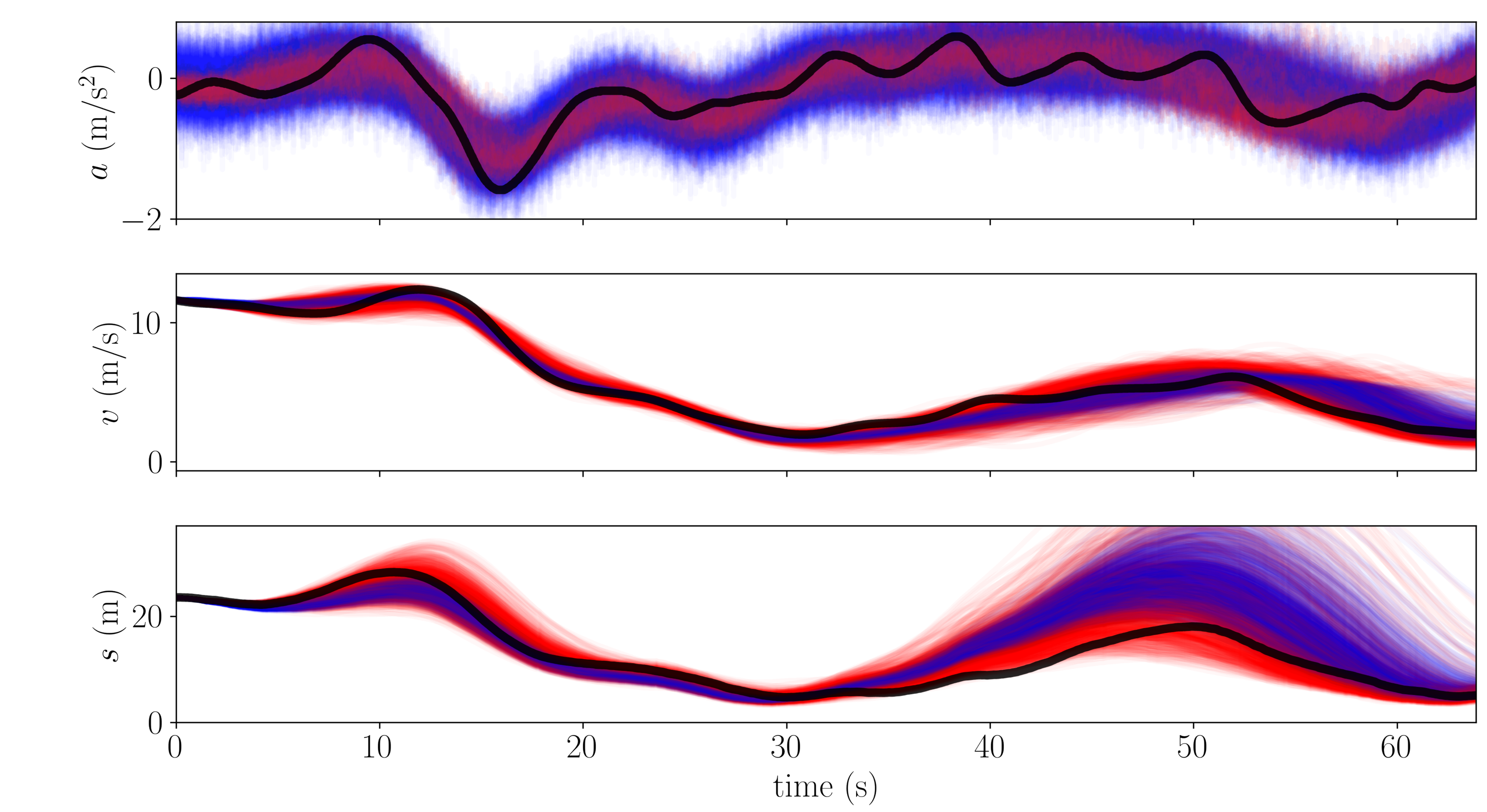
(a) Hierarchical B-IDM posteriors. (b) Hierarchical MA-IDM posteriors.

Fig. 4: The posterior distributions and the correlation matrices of the parameters of a truck driver in the hierarchical B-IDM and MA-IDM.

Deterministic and Stochastic Simulations



(a) Deterministic simulations (hierarchical model).



(b) Stochastic simulations (hierarchical model).

Fig. 5: The deterministic (top) and stochastic (bottom) simulation results of a truck driver. The black lines are the ground truth driving data; The yellow and fuchsia dotted lines are the predicted motion states with the expectations of parameter posteriors; The red and blue lines are the predicted motion states with the parameter samples drawn from posteriors.

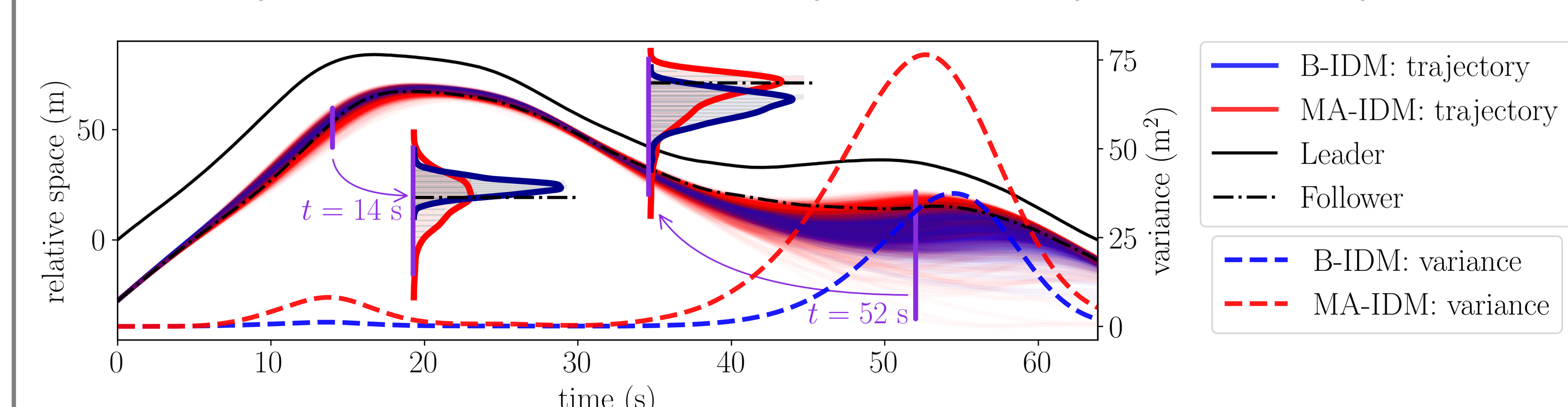


Fig. 6: The time-space diagram of the follower's posterior trajectories at the point view of another 'observing' vehicle with a constant mean speed.