Stochastic Modeling and Simulations of Car-Following Behaviors

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About me

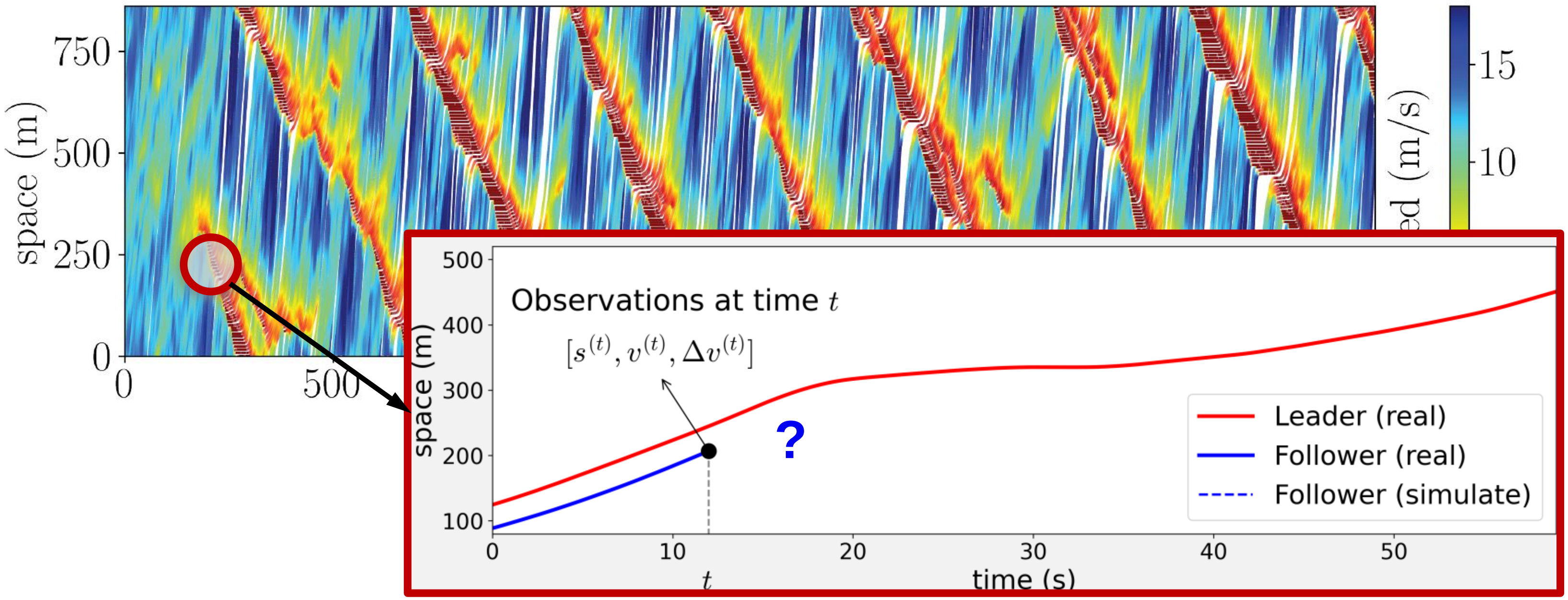
- Smart Transportation Lab at McGill University

- Research interests:
    - **Traffic flow theory;**
    - **Traffic Simulation;**
    - **Human driving behavior;**
    - **Bayesian learning;**
    - **Multi-agent interaction modeling.**

- I aim to bridge the gap between **theoretical modeling** and **practical traffic simulation** using advanced **statistical techniques**. Driven by a passion for understanding human driving behavior, my work seeks to enhance microscopic traffic simulations, ultimately contributing to safer and more efficient transportation systems.

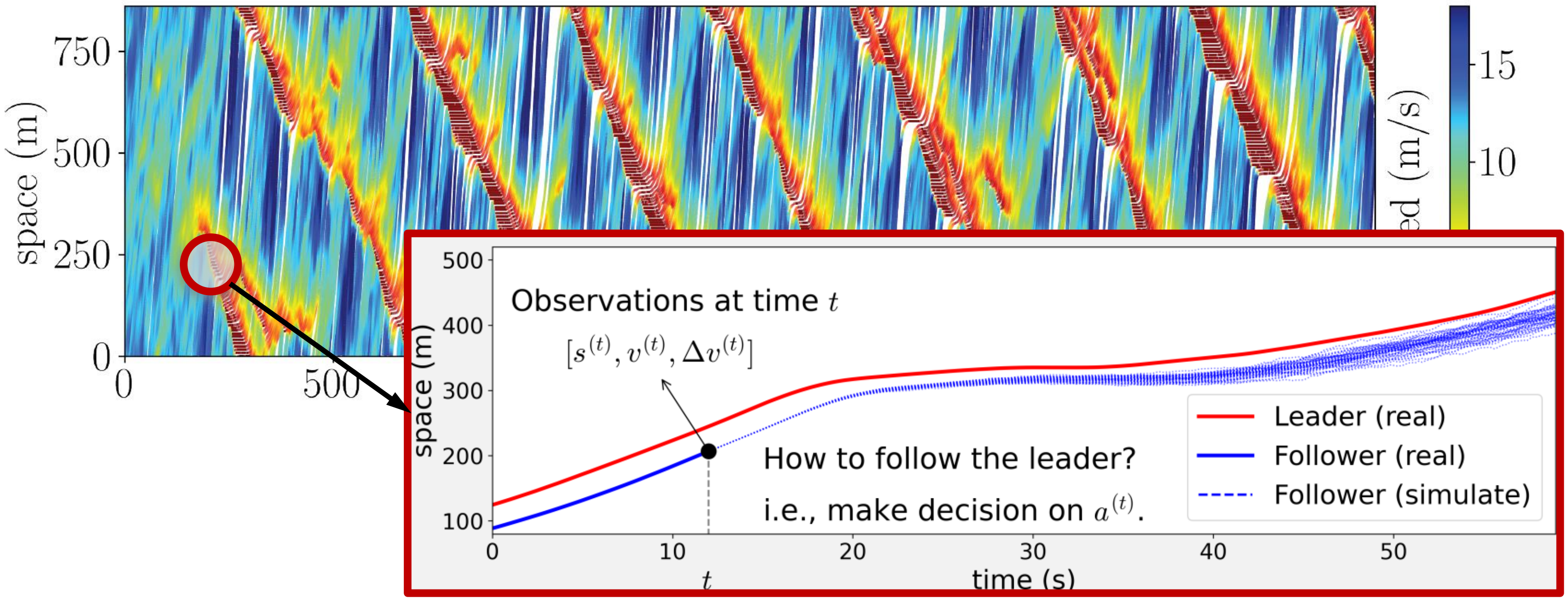
Motivation / background

- How would the vehicle react in response to the leading vehicle?



Motivation / background

- How would the vehicle react in response to the leading vehicle?

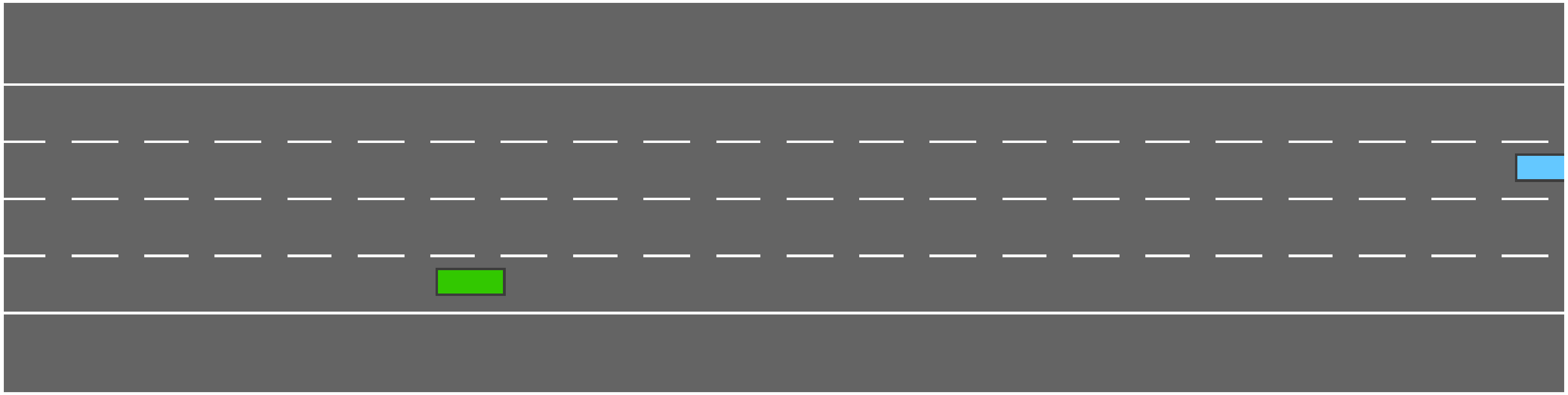


- What do we need for traffic simulations?

Motivation / background

- The goal of traffic simulations:
  - **Past**: reproduce traffic phenomenon.
  - **Future**: support the development and test of control algorithms:
    - Connected and Automated Vehicle;
    - Reinforcement learning for traffic control/management;
    - Human drivers still involved;
    - Safety, predictability, and uncertainty;

(How well are the blue cars performing in this simulator?)



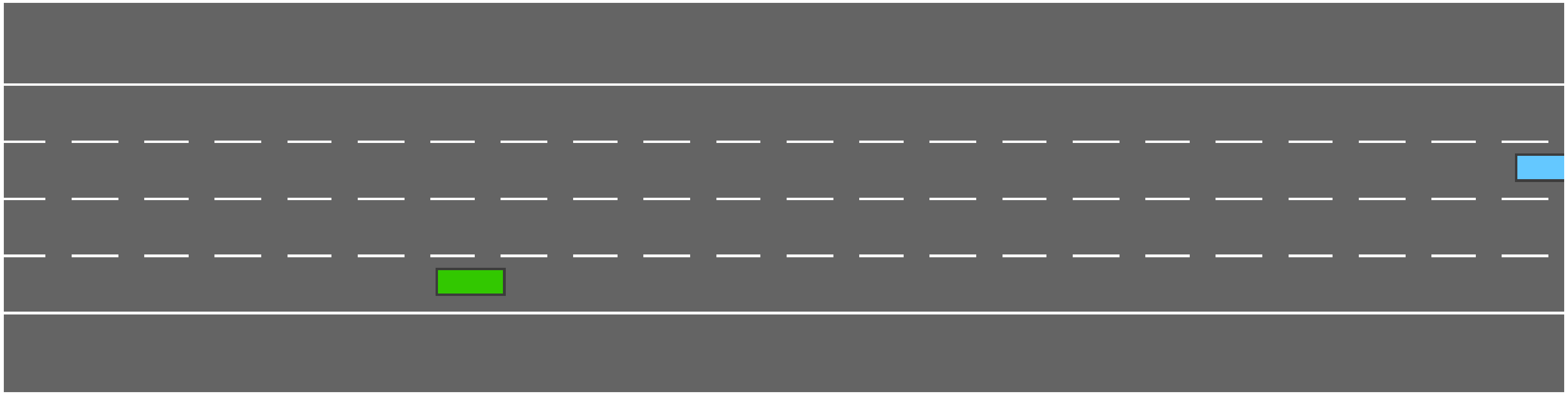
Motivation / background

- **A realistic traffic simulator is not just a convenience but a necessity.**

- How do we introduce **realistic randomness**?
  - x Deterministic car-following models;
  - ✓ **Probabilistic** car-following models with **uncertainty quantification**.

In this presentation, we are interested in:

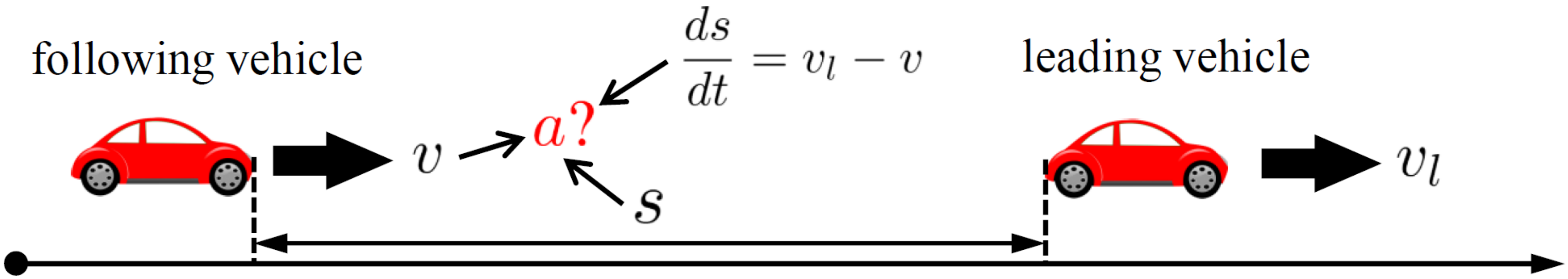
- ➢ How do we model the human-driver car-following behaviors?
- ➢ How do we simulate human-like car-following behaviors?



Outline

- **Background**
  - Temporal correlations in driving behaviors
  - The general form of car-following models
- **Probabilistic Modeling Framework**
  - Stationary: IDM (mean model) + stochastic processes (GP)
  - Nonstationary: NN (mean model) + stochastic processes (GP)
- **Stochastic Simulation**
  - Short-term single car-following pair
  - Long-term multiple car-following pairs
- **Discussions**

Intelligent driver model



- **Intelligent Driver Model (IDM)** (Treiber et al. 2000)

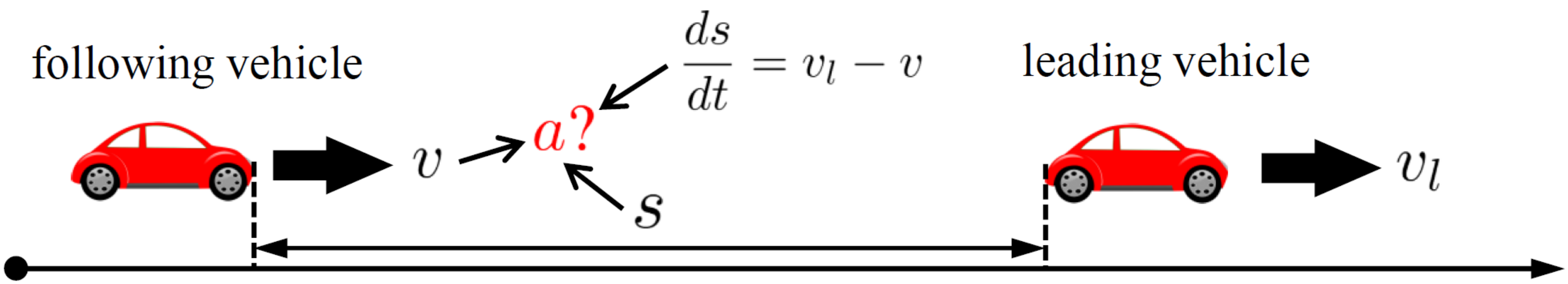
$$a_{\mathrm{IDM}} = \alpha \left(\underbrace{\boxed{1 - \left(\frac{v}{v_0}\right)^{\delta}}}_{\text{free-flow}} - \underbrace{\boxed{\left(\frac{s^*(v,\Delta v)}{s}\right)^2}}_{\text{interaction}} \right)$$

$$s^*(v,\Delta v) = s_0 + s_1\sqrt{\frac{v}{v_0}} + vT + \frac{v\,\Delta v}{2\sqrt{\alpha\,\beta}}$$

- v_0: desired speed;
- s_0: jam spacing;
- T: time headway;
- α: maximum acceleration;
- β: comfortable deceleration rate.

$$\boldsymbol{\theta} = \underbrace{[v_0, s_0, T, \alpha, \beta]}_{\text{parameter set}}$$

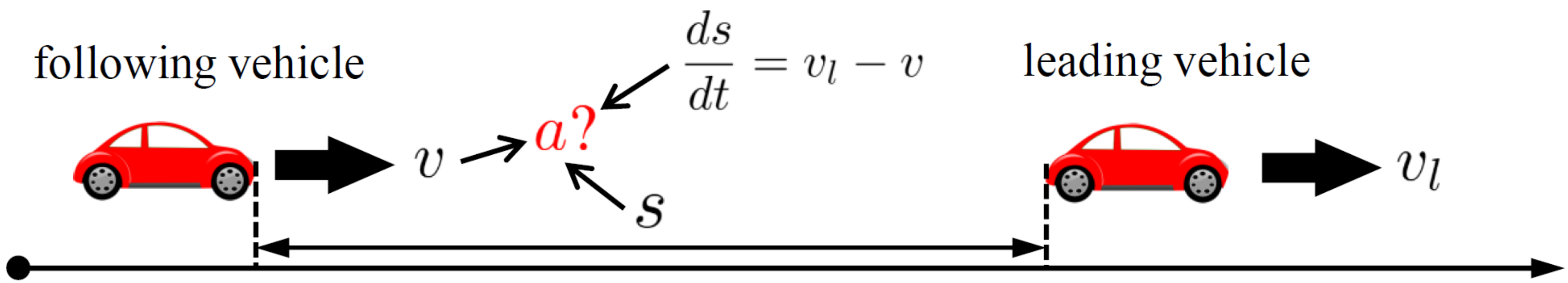
Intelligent driver model



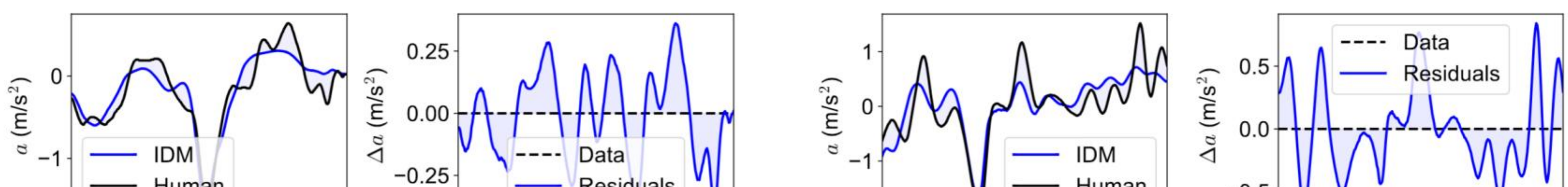
- Gaussian IDM assumes: $a^{(t)} \approx a_{\mathrm{IDM}}^{(t)}$. ➡ Likelihood: $\mathcal{N}(\hat{a}^{(t)} | a_{\mathrm{IDM}}^{(t)}, \sigma_\epsilon^2)$
- Calibration by MLE: $\max_{\boldsymbol{\theta}} \prod_{t=1}^{T} \text{likelihood}$ and $\boldsymbol{\theta} = \underbrace{[v_0, s_0, T, \alpha, \beta]}_{\text{parameter set}}$
- Loss function in literature (Punzo et al. 2021):

$$\min_{\boldsymbol{\theta}} \frac{1}{T} \sum_{t=1}^{T} \underbrace{(a_{\mathrm{IDM}}^{(t)} - \hat{a}^{(t)})^2}_{\text{acceleration}} + \frac{\alpha}{T} \sum_{t=1}^{T} \underbrace{(v_{\mathrm{IDM}}^{(t)} - \hat{v}^{(t)})^2}_{\text{speed}} + \frac{\beta}{T} \sum_{t=1}^{T} \underbrace{(x_{\mathrm{IDM}}^{(t)} - \hat{x}^{(t)})^2}_{\text{position}}$$

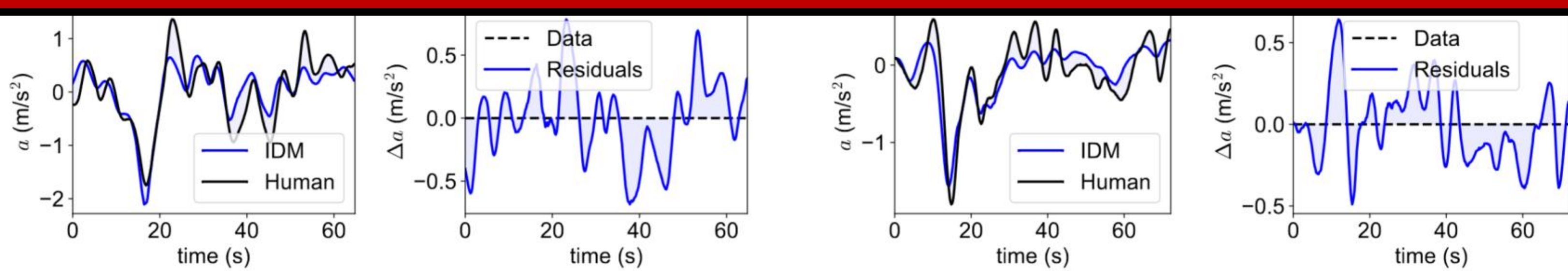
Intelligent driver model



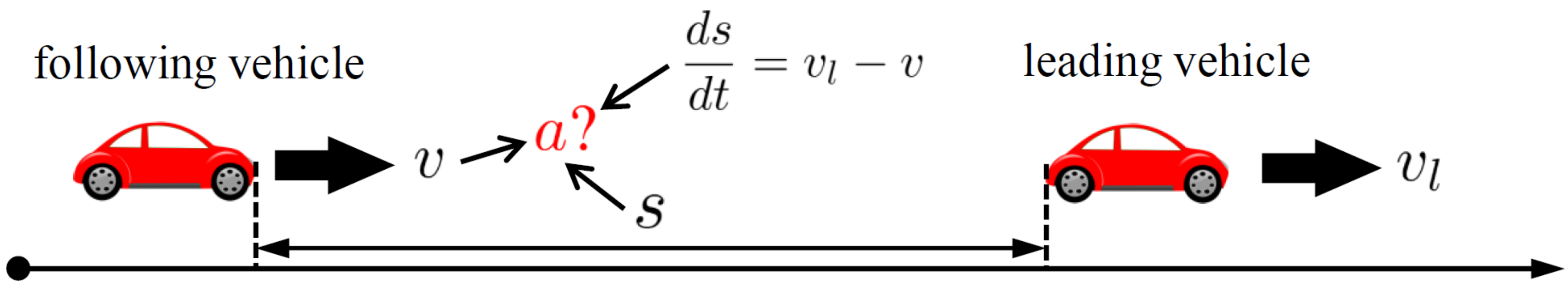
- IDM assumes: $a^{(t)} \approx a^{(t)}_{\mathrm{IDM}}$. Let's visualize the residuals $a^{(t)} - a^{(t)}_{\mathrm{IDM}}$.



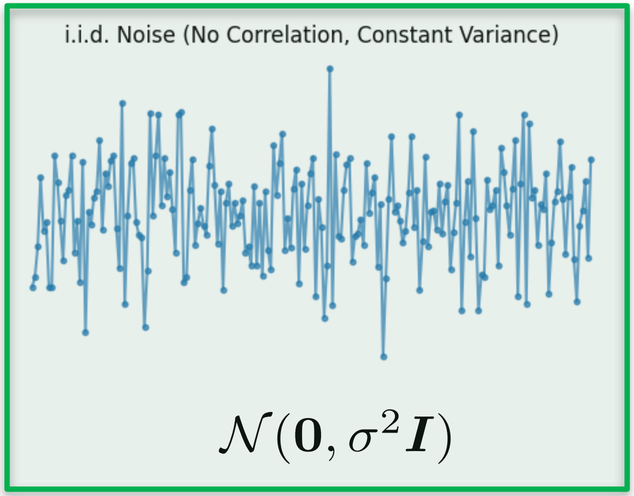
IDM captures much information,
but ***some are still left in the residuals***!

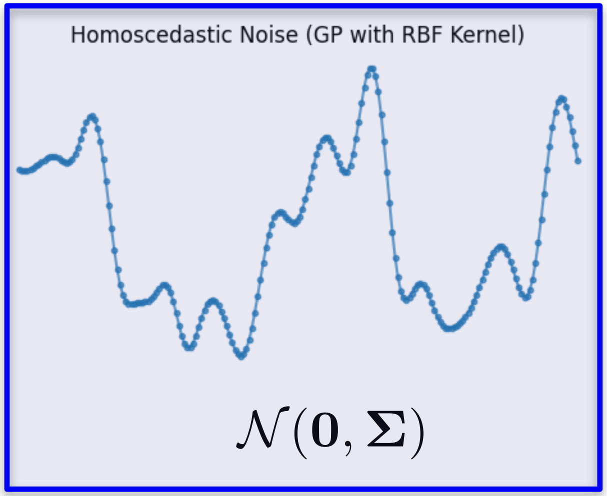


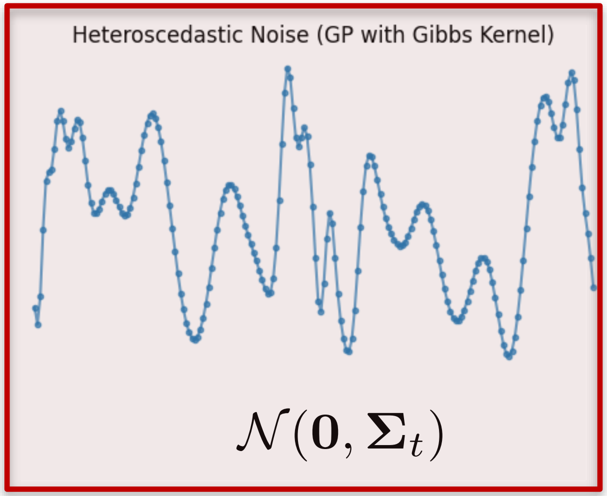
Temporal correlations in driving behaviors



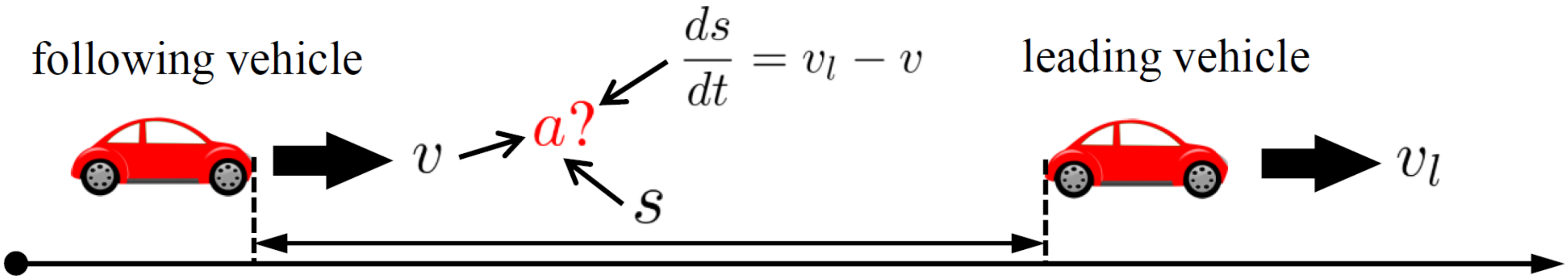
- For a human-driver CF model, what do we miss?
  - **Temporally correlated errors**
  - **Heteroscedasticity**







Temporal correlations in driving behaviors



- For a human-driver CF model, what do we miss?
  - **Temporally correlated errors**
  - **Heteroscedasticity**

Core contribution:

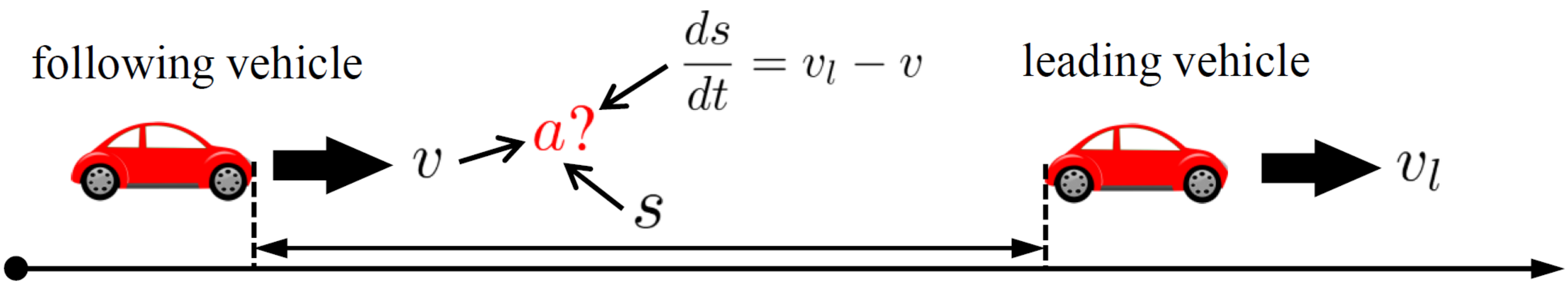
temporal correlations (captures how past behaviors influence future behaviors)

- Therefore, we assume $a(x,t) \approx \boxed{f_{\mathrm{CFM}}(x;\boldsymbol{\theta})} + \boxed{\delta(t)}$ (Zhang et al. 2025)

mean car-following model

Inspired by GLS, see my post From Ordinary Least Squares (OLS) to Generalized Least Squares (GLS)

The general form of car-following models



- We assume: $a(x,t) \approx \boxed{f_{\mathrm{CFM}}(x;\boldsymbol{\theta})} + \boxed{\delta(t)}$ (Zhang et al. 2025)

- IDM assumes: $a(x,t) \approx \boxed{a_{\mathrm{IDM}}(x;\boldsymbol{\theta})}.$ (Treiber et al. 2000)

<u>Missed the temporal part</u> $\boxed{\delta(t)}$

TO-DO:

- Consider $\delta(t)$ in modeling;
- Model $\delta(t+1)|\delta(t)$ in simulation.

Memory-Augmented IDM (MA-IDM) (Zhang and Sun 2024)

How to model $\delta(t)$ and $\delta(t+1)|\delta(t)$?

- We assume:

$$a(x,t) = \boxed{f_{\text{CFM}}(x;\boldsymbol{\theta})} + \boxed{\delta(t)} + \boxed{\epsilon_t,} \ \epsilon_t \overset{i.i.d.}{\sim} \mathcal{N}(0,\sigma_\epsilon^2)$$

- Bayesian IDM assumes:

$$a^{(t)} = \boxed{a^{(t)}_{\text{IDM}}} + \boxed{\epsilon_t, \ \epsilon_t \overset{i.i.d.}{\sim} \mathcal{N}(0,\sigma_\epsilon^2)} \quad \Rightarrow \boldsymbol{a}|\boldsymbol{i},\boldsymbol{\theta} \sim \mathcal{N}(\boxed{\boldsymbol{a}_{\text{IDM}}}, \boxed{\sigma_\epsilon^2 \boldsymbol{I}})$$

incorrect

Vector form with Multivariate Normal

- MA-IDM assumes:

$$a^{(t)} = \boxed{a^{(t)}_{\text{IDM}}} + \boxed{a^{(t)}_{\text{GP}}} + \boxed{\epsilon_t} \quad \Rightarrow \boldsymbol{a}|\boldsymbol{i},\boldsymbol{\theta} \sim \mathcal{N}(\boxed{\boldsymbol{a}_{\text{IDM}}}, \boxed{\boldsymbol{K}} + \boxed{\sigma_\epsilon^2 \boldsymbol{I}})$$

mean model residuals *i.i.d.* error

where ***K*** is a kernel matrix .

[Chengyuan Zhang and Lijun Sun. (2024). Bayesian calibration of the intelligent driver model. *IEEE Transactions on Intelligent Transportation Systems*.]

Memory-Augmented IDM (MA-IDM) (Zhang and Sun 2024)

- Bayesian IDM assumes:

$$a^{(t)} = a^{(t)}_{\text{IDM}} + \epsilon_t,\ \ \epsilon_t \overset{i.i.d.}{\sim} \mathcal{N}(0, \sigma_\epsilon^2) \qquad \Rightarrow \boldsymbol{a}|\boldsymbol{i}, \boldsymbol{\theta} \sim \mathcal{N}(\boldsymbol{a}_{\text{IDM}}, \sigma_\epsilon^2 \boldsymbol{I})$$

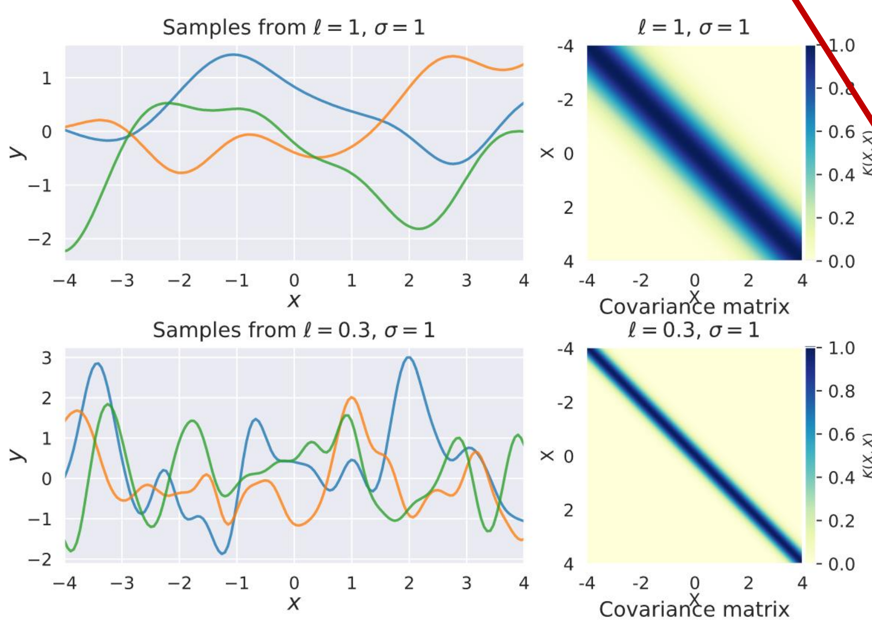
- MA-IDM assumes:

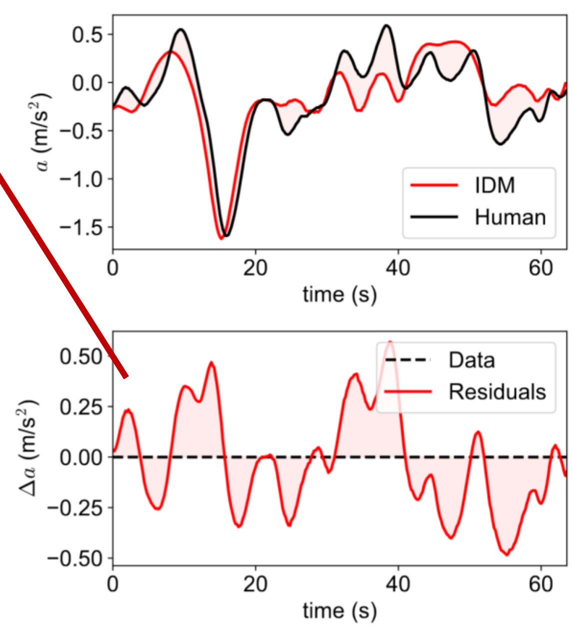
$$a^{(t)} = a^{(t)}_{\text{IDM}} + \boxed{a^{(t)}_{\text{GP}}} + \epsilon_t \qquad \Rightarrow \boldsymbol{a}|\boldsymbol{i}, \boldsymbol{\theta} \sim \mathcal{N}(\boldsymbol{a}_{\text{IDM}}, \boxed{\boldsymbol{K}} + \sigma_\epsilon^2 \boldsymbol{I})$$

residuals

where ***K*** is a kernel matrix .

- Gaussian processes

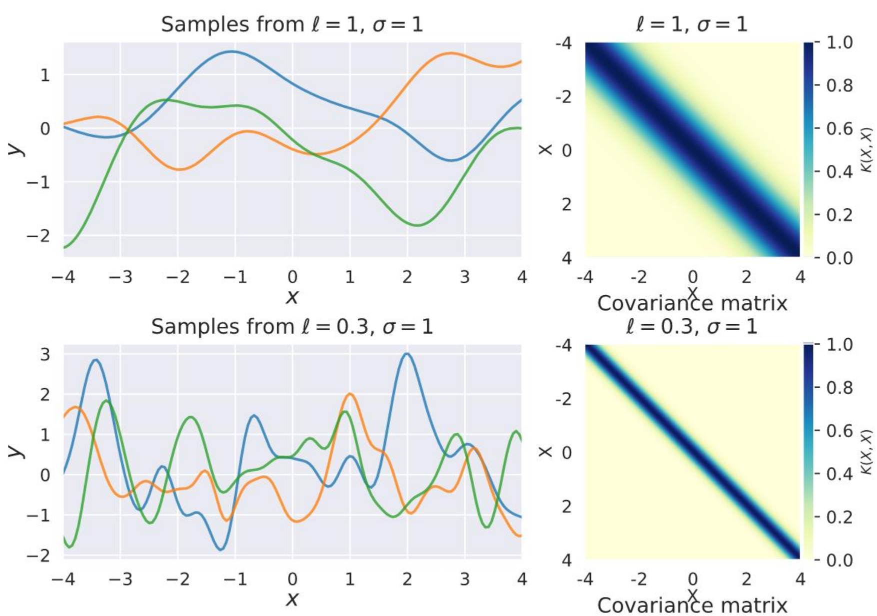


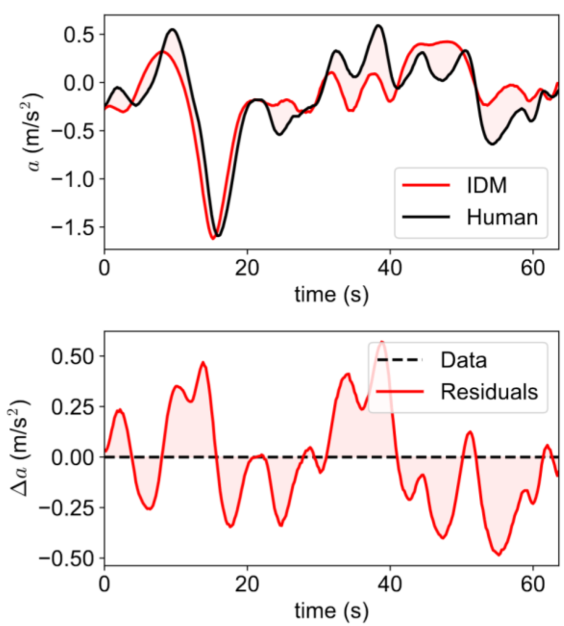


Memory-Augmented IDM (MA-IDM) (Zhang and Sun 2024)

Construct the kernel matrix *K*:

- SE kernel: $$k_{\mathrm{SE}}(t, t'; \boldsymbol{\lambda}) := \sigma^2 \exp\left(-\frac{d(t,t')^2}{2\ell^2}\right),$$
- Matern kernel: $$k^{\nu}_{\mathrm{Matérn}}(t, t'; \boldsymbol{\lambda}) := \sigma^2 \frac{2^{1-\nu}}{\Gamma(\nu)} \left(\sqrt{2\nu}\frac{d(t,t')}{\ell}\right)^{\nu} K_{\nu}\left(\sqrt{2\nu}\frac{d(t,t')}{\ell}\right),$$
- Gaussian processes



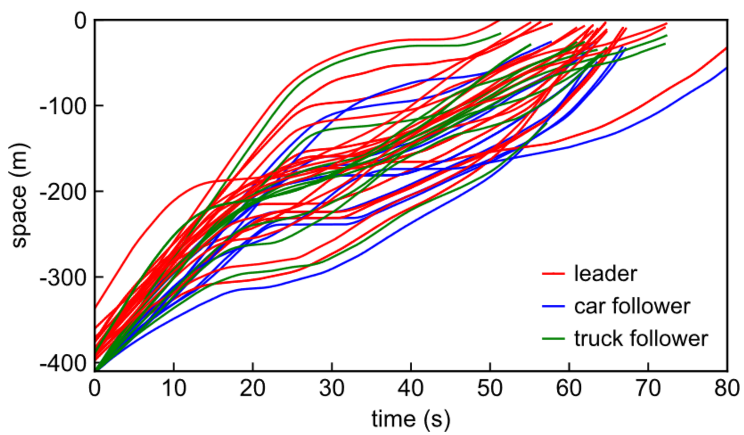


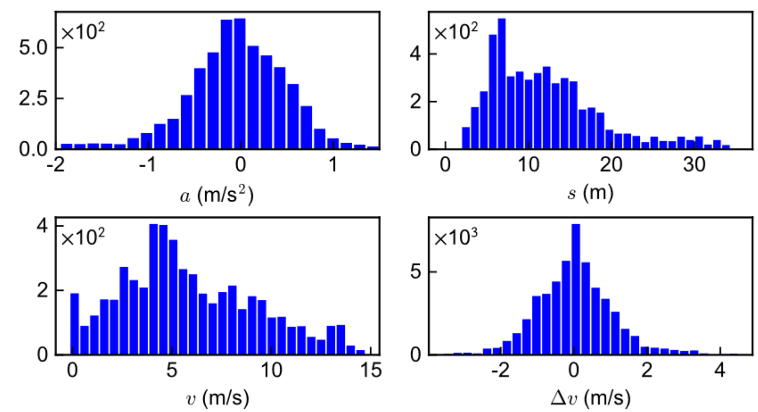
Experiments – Car-Following Data Extraction

- **HighD** dataset: (Krajewski et al. 2018)

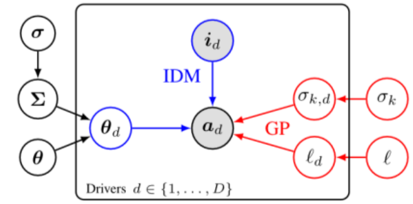
https://levelxdata.com/highd-dataset/

- naturalistic vehicle trajectories → leader-follower pairs.

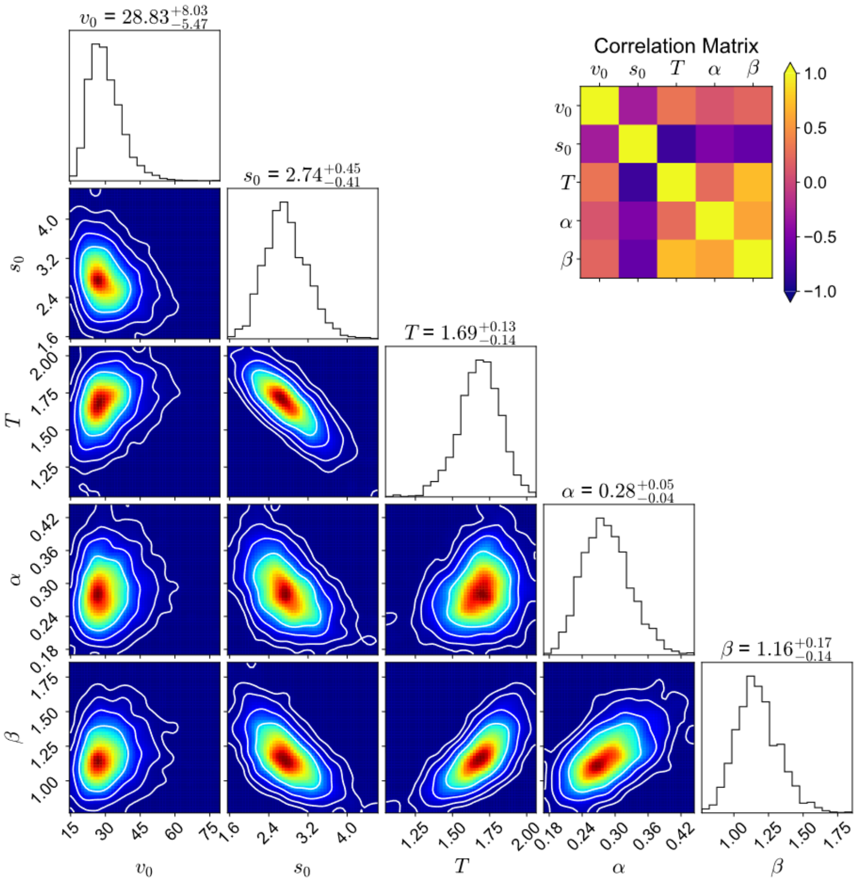




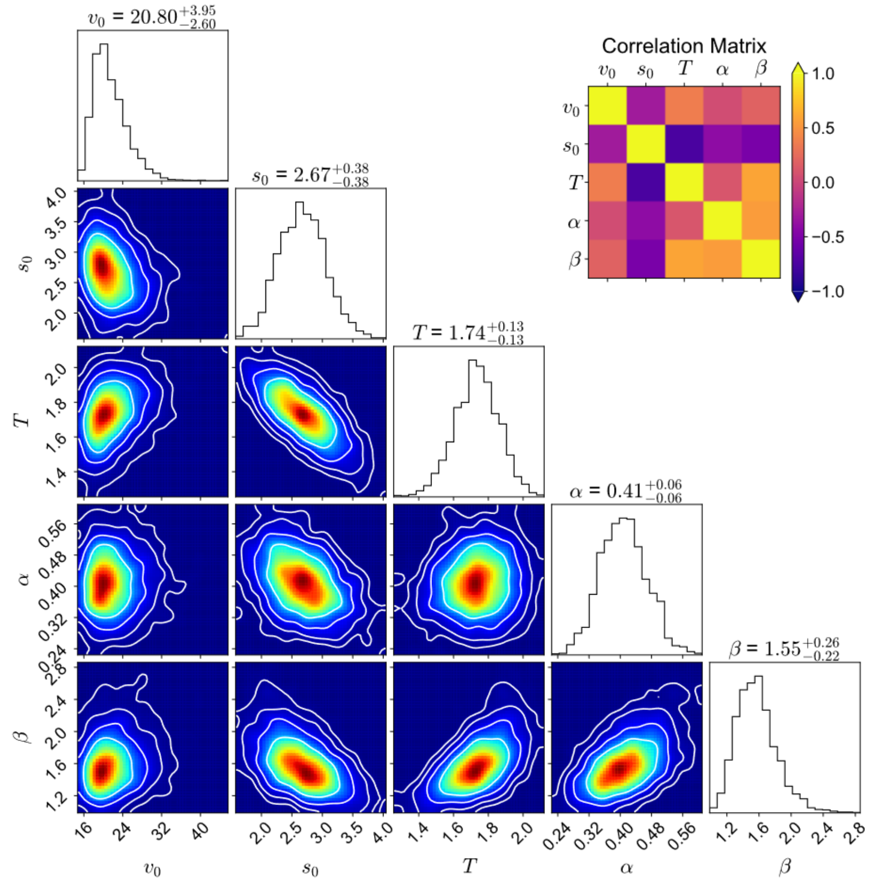
Experiments – Identified IDM Parameters



Similar posterior distribution shape but **more concentrated**



(a) Hierarchical B-IDM posteriors of truck #211.



(b) Hierarchical MA-IDM posteriors of truck #211.

We can draw samples (IDM parameters) from the posterior distributions!!

Simulations – Deterministic *v.s.* Stochastic

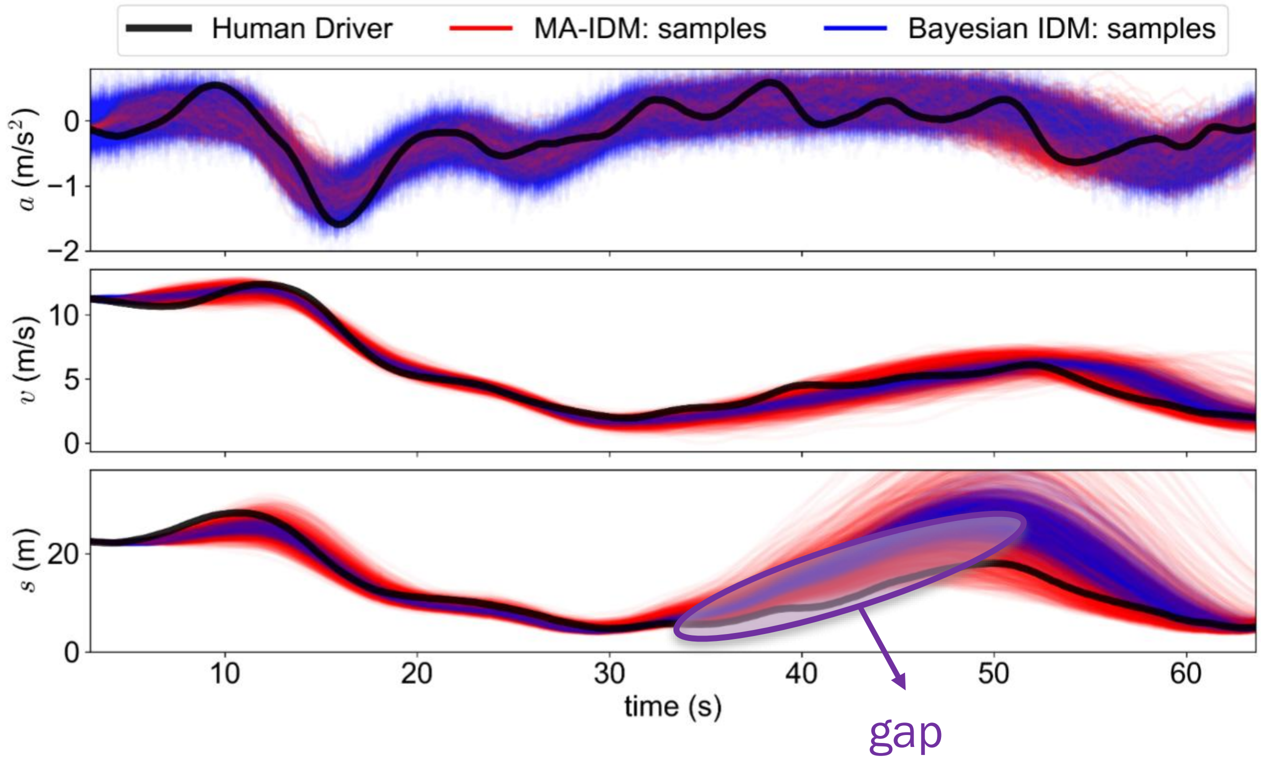
- **B-IDM assume:** $a_d^{(t)} \approx \boxed{a_{\mathrm{IDM},d}^{(t)}}$ $\quad \boldsymbol{a} \sim \mathcal{N}(\boldsymbol{a}_{\mathrm{IDM}}, \sigma_\epsilon^2 \boldsymbol{I})$

- **MA-IDM assumes:** $a_d^{(t)} \approx \boxed{a_{\mathrm{IDM},d}^{(t)}} + \boxed{a_{\mathrm{GP},d}^{(t)}}$ $\quad \boldsymbol{a} \sim \mathcal{N}(\boldsymbol{a}_{\mathrm{IDM}}, \boldsymbol{K} + \sigma_\epsilon^2 \boldsymbol{I})$

- **Stochastic simulation for step *t* :**

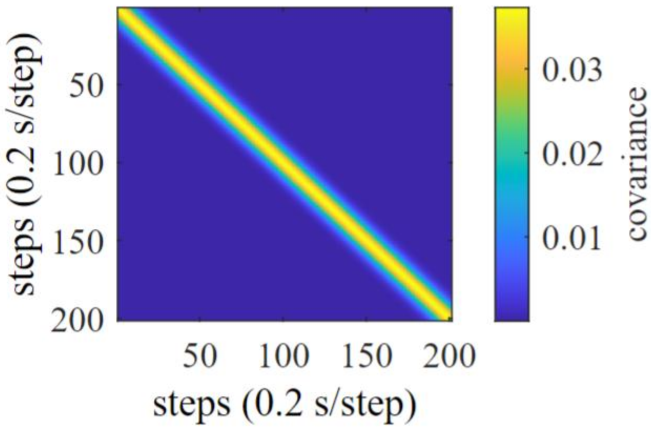
  1) Obtain the first term $\boxed{a_{\mathrm{IDM},d}^{(t)}}$ by feeding $\boldsymbol{\theta}_d$ and inputs into the IDM function;
  2) Draw a sample $\boxed{a_{\mathrm{GP},d}^{(t)} | \boldsymbol{a}_{\mathrm{GP},d}^{(t-T:t-1)}}$ at time t from the GP to obtain the temporally correlated information $a_{\mathrm{GP},d}^{(t)}$;

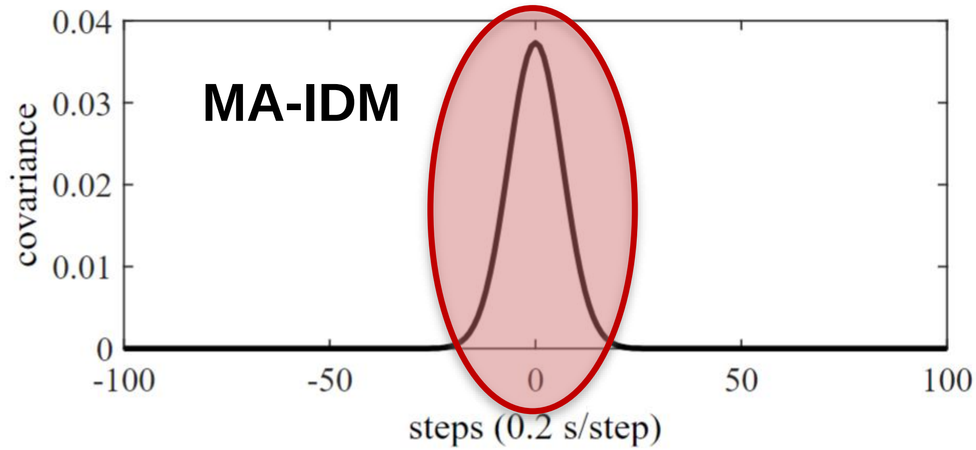
Simulations – Stochastic Simulation (MA-IDM v.s. B-IDM)



- **Action uncertainty is scenario specific**: When the leading vehicle is braking, all drivers must decelerate; But when the leading vehicle accelerates, actions are more uncertain at their own will.
- MA-IDM has a better calibration result than B-IDM. Even B-IDM is with a large noise variance, it still cannot bridge the gap (*i.e.*, with bad uncertainty quantification.)

Stationary kernel and nonstationary behaviors





The lengthscale is about 1.5 sec → capture correlations within 4~5 sec (3-sigma in Normal distribution).

But it assumes that the temporal correlations are stationary.

- Stationary temporal correlations: The correlations between time steps are assumed to be **constant** over time.
- Human driving behavior is **dynamic**. Drivers might react differently under congested traffic conditions compared to open road driving.

Nonstationary temporal correlations (Zhang et al. 2025)

- **We assume:**

$$a(x,t) = \boxed{f_{\mathrm{CFM}}(x;\boldsymbol{\theta})} + \boxed{\delta(t)} + \boxed{\epsilon_t,}\ \epsilon_t \overset{i.i.d.}{\sim} \mathcal{N}(0,\sigma_\epsilon^2)$$

- **MA-IDM assumes:**

$$a^{(t)} = \boxed{a_{\mathrm{IDM}}^{(t)}} + \boxed{a_{\mathrm{GP}}^{(t)}} + \boxed{\epsilon_t} \quad \Rightarrow \boldsymbol{a}|\boldsymbol{i},\boldsymbol{\theta} \sim \mathcal{N}(\boxed{\boldsymbol{a}_{\mathrm{IDM}}}, \boxed{\boldsymbol{K}} + \boxed{\sigma_\epsilon^2 \boldsymbol{I}})$$

Homoscedasticity assumption with a stationary kernel. (inappropriate)

- **Nonstationary model assumes:**

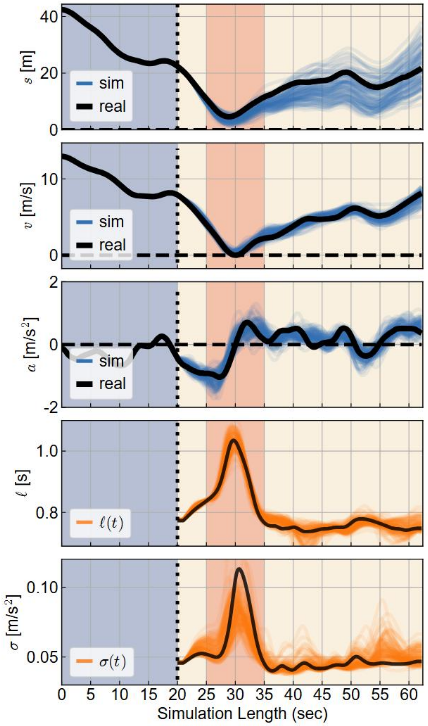
$$a^{(t)} = \boxed{a_{\mathrm{NN}}^{(t)}} + \boxed{a_{\mathrm{GP}}^{(t)}} + \boxed{\epsilon_t} \quad \Rightarrow \boldsymbol{a}|\boldsymbol{i},\boldsymbol{\theta}_{\mathrm{NN}} \sim \mathcal{N}(\boxed{\boldsymbol{a}_{\mathrm{NN}}}, \boxed{\boldsymbol{K}} + \boxed{\sigma_\epsilon^2 \boldsymbol{I}})$$

Heteroscedasticity assumption with a nonstationary kernel (Gibbs kernel)

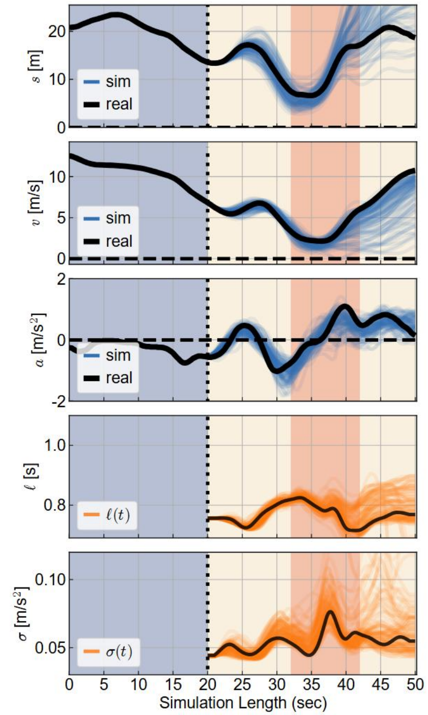
$$k_{\mathrm{Gibbs}}(t,t';\boldsymbol{\lambda}) := \sigma(t)\sigma(t')\sqrt{\frac{2\ell(t)\ell(t')}{\ell(t)^2+\ell(t')^2}}\exp\left(-\frac{(t-t')^2}{\ell(t)^2+\ell(t')^2}\right)$$

[Chengyuan Zhang, Zhengbing He, Cathy Wu, and Lijun Sun. (2025). Stochastic Modeling of Car-Following Behaviors with Nonstationary Temporal Correlations. *Preprint* (*under review*).]

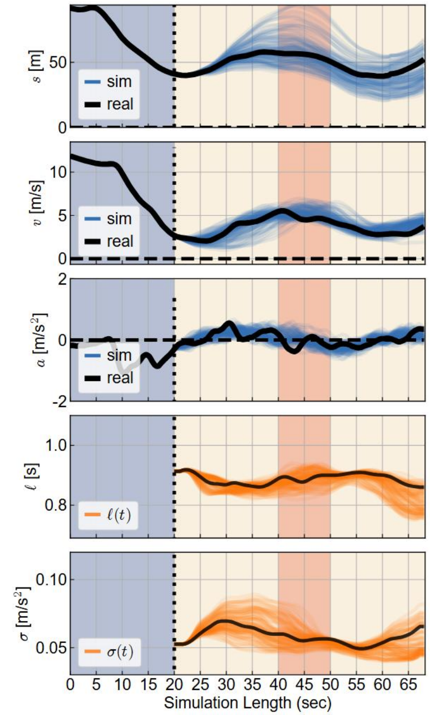
Nonstationary temporal correlations (Zhang et al. 2025)



(a) Car-following Pair #1.



(b) Car-following Pair #2.



(c) Car-following Pair #3.

Lengthscale:

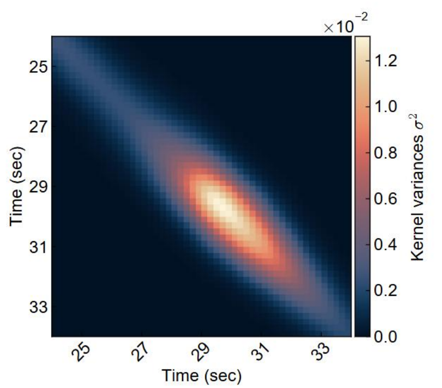
Smooth driving ↑

Abrupt transition ↓

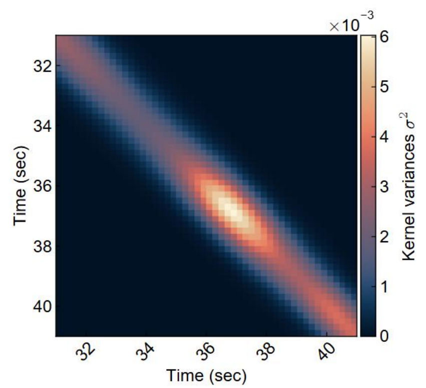
Kernel variance:

Free/steady ↑

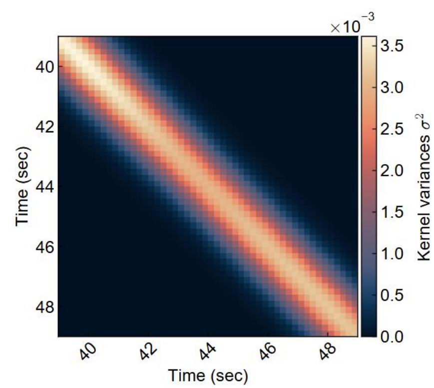
Safety-critical ↓



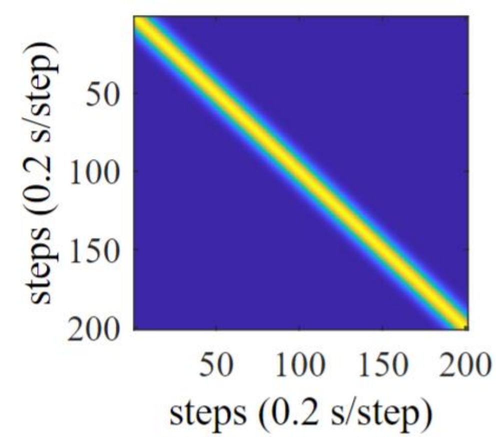
(d) Example $\boldsymbol{K}$ in Pair #1.



(e) Example $\boldsymbol{K}$ in Pair #2.



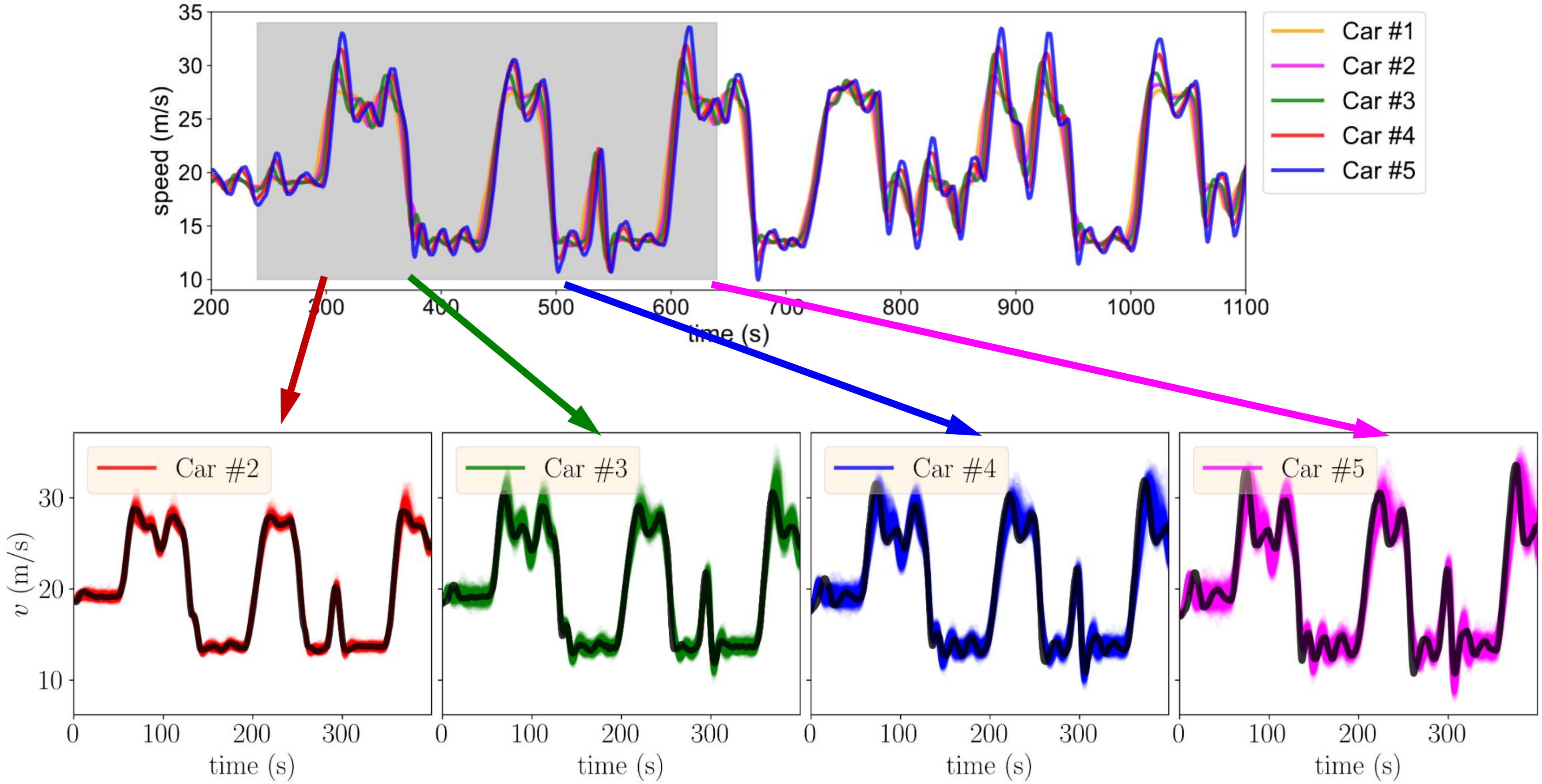
(f) Example $\boldsymbol{K}$ in Pair #3.



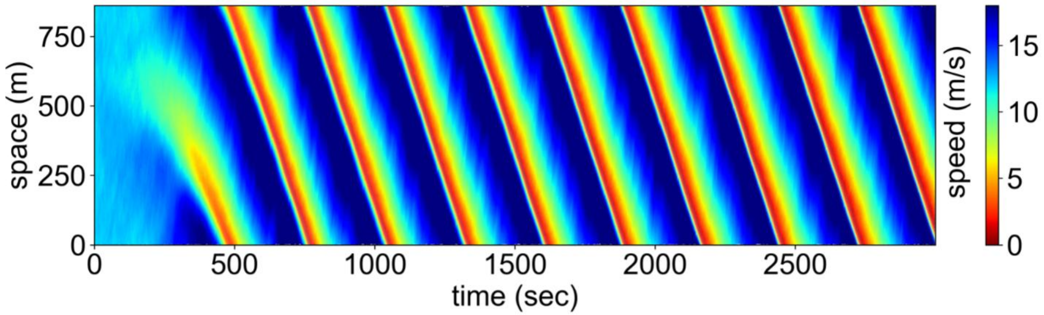
Simulations – Multi-vehicle scenario: Platoon

OpenACC dataset

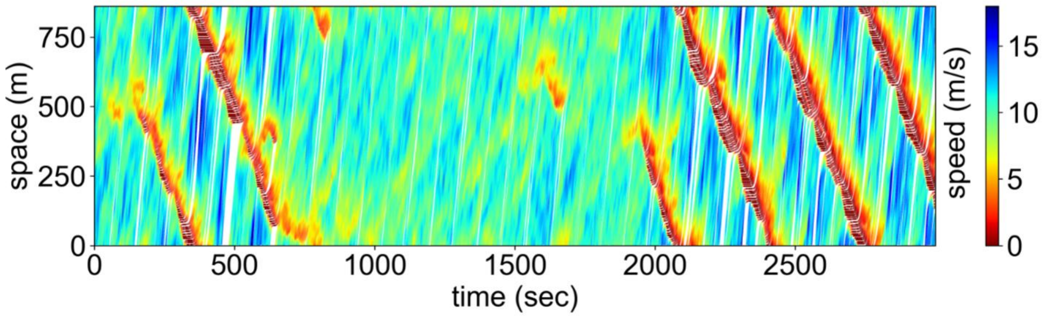
http://data.europa.eu/89h/9702c950-c80f-4d2f-982f-44d06ea0009f



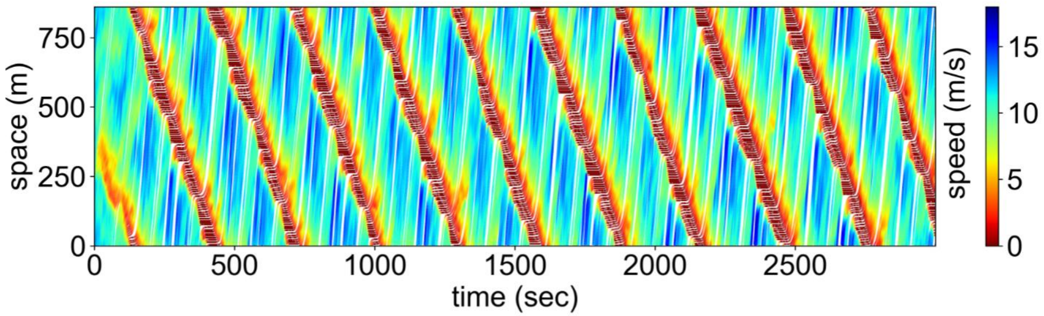
Simulations – Multi-vehicle scenario: Ring road



(a) Simulation with fixed IDM parameters and random white noise.



(b) Light traffic simulation with dynamic IDM ($p = 4$).



(c) Dense traffic simulation with dynamic IDM ($p = 4$).

Sugiyama experiment

General Overview

$$a(x,t) = f_{\mathrm{CFM}}(x;\boldsymbol{\theta}) + \delta(t) + \epsilon_t, \quad \epsilon_t \overset{i.i.d.}{\sim} \mathcal{N}(0,\sigma_\epsilon^2)$$

Table 1: Modeling of temporal correlations in the literature.

Reference	$f_{\mathrm{CFM}}(\boldsymbol{x})$	$\delta(t)$	Nonstationary?
Treiber et al. (2006)	IDM	Ornstein-Uhlenbeck (OU) processes	✗
Hoogendoorn and Hoogendoorn (2010)	GHR/IDM	Cochrane-Orcutt correction (i.e., AR(1) process)	✗
Zhang and Sun (2024)	IDM	Gaussian processes (GPs)	✗
Zhang et al. (2024a)	IDM	AR processes with higher orders	✗
Zhang et al. (2025)	NN	nonstationary GPs	✓

- The GP with a Matérn 1/2 kernel can be seen as the continuous-time counterpart of the discrete-time AR(1) process; The AR(1) process can be seen as a discrete-time analog of the Matérn 1/2 kernel.
- The OU process is a continuous-time counterpart to the AR(1) process. The OU process is equivalent to the GP with a Matérn 1/2 kernel.
- The AR(1) process can be considered a discrete-time version of the OU process. Both processes have exponential autocorrelation functions, but the AR(1) process is defined in discrete time, while the OU process is defined in continuous time.
- the Cochrane-Orcutt correction is a method for addressing autocorrelation in regression models, assuming an AR(1) structure

Discussion and takeaway

Human-like car-following behaviors:

$$a(x,t) = \boxed{f_{\mathrm{CFM}}(x;\boldsymbol{\theta})} + \boxed{\delta(t)} + \boxed{\epsilon_t,}\ \epsilon_t \overset{i.i.d.}{\sim} \mathcal{N}(0,\sigma_\epsilon^2)$$

- **Modeling: Significance of appropriate uncertainty quantification!**
- **Simulation: Significance of stochastic simulation!**

- **Inappropriate assumptions and solutions:**
  - Independent and identically distributed (i.i.d.) errors; (GP/AR)
  - Homoscedasticity (constant variance of errors); (nonstationary GP)

References

- Treiber, M., Hennecke, A., & Helbing, D. (2000). Congested traffic states in empirical observations and microscopic simulations. Physical Review E, 62(2), 1805.
- Punzo, V., Zheng, Z., & Montanino, M. (2021). About calibration of car-following dynamics of automated and human-driven vehicles: Methodology, guidelines and codes. Transportation Research Part C: Emerging Technologies, 128, 103165.
- Krajewski, R., Bock, J., Kloeker, L., & Eckstein, L. (2018). The highd dataset: A drone dataset of naturalistic vehicle trajectories on german highways for validation of highly automated driving systems. In 2018 21st International Conference on Intelligent Transportation Systems (ITSC) (pp. 2118-2125). IEEE.
- Anesiadou, A., Makridis, M., Ciuffo, B., & Mattas, K. (2020): Open ACC Database. European Commission, Joint Research Centre (JRC) [Dataset] PID: http://data.europa.eu/89h/9702c950-c80f-4d2f-982f-44d06ea0009f
- Treiber, M., Kesting, A., & Helbing, D. (2006). Delays, inaccuracies and anticipation in microscopic traffic models. Physica A: Statistical Mechanics and its Applications, 360(1), 71-88.

Read More

➢ **Paper:**

Zhang, C., & Sun, L. (2024). Bayesian calibration of the intelligent driver model. *IEEE Transactions on Intelligent Transportation Systems*. (IDM with GP)

Zhang, C., Wang, W., & Sun, L. (2024). Calibrating car-following models via Bayesian dynamic regression. *Transportation Research Part C: Emerging Technologies, 104719. (ISTTT25 Special Issue)*. (IDM with AR)

Zhang, C., Zheng, H., Wu, C., & Sun, L. (2025). Stochastic Modeling of Car-Following Behaviors with Nonstationary Temporal Correlations. *Preprint (under review)*. (NN with nonstationary GP)

➢ **Code:**

https://github.com/Chengyuan-Zhang/IDM_Bayesian_Calibration

Thanks!
Questions?

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