From Micro Interactions to Traffic Flow: Stochastic Driver Models for Realistic Traffic Simulation

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MIT Wu Lab (online)

About me

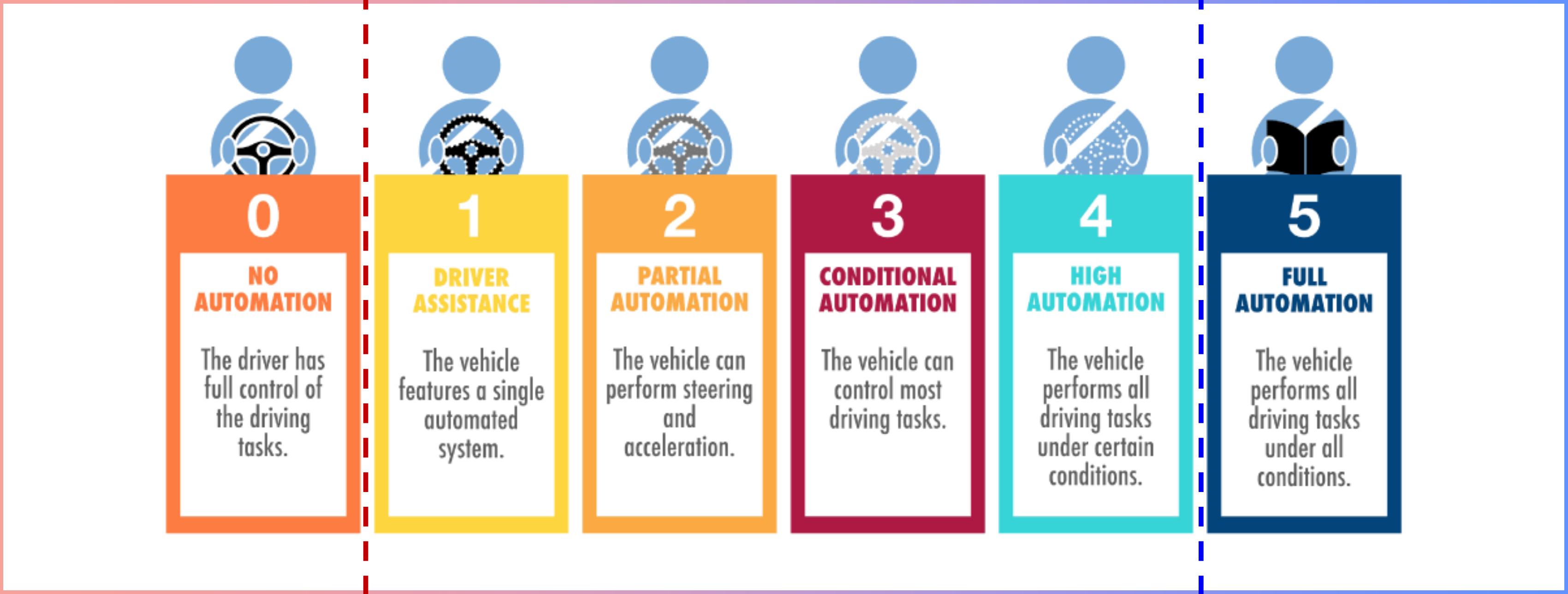
- Smart Transportation Lab at McGill University

- Research interests:
  - Traffic flow theory;
  - Stochastic simulation;
  - Human behavior modeling;
  - Bayesian learning;
  - Multi-agent interaction.

- My research focuses on **Bayesian inference**, **spatiotemporal modeling**, **traffic flow theory**, and **multi-agent interaction modeling** within **intelligent transportation systems**, with an emphasis on bridging the gap between theoretical modeling and practical traffic simulation through advanced statistical techniques with appropriate **uncertainty quantification**.
- My motivation lies in advancing the understanding of **human driving behaviors** to improve **microscopic traffic simulations**, ultimately contributing to safer and more efficient transportation systems.

Motivation

- **Levels of Autonomous Driving (AD)**



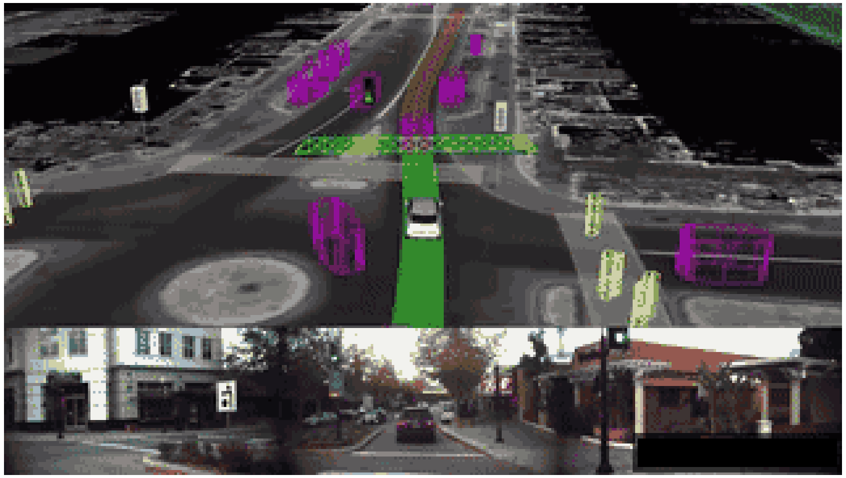
Human Driving | **Mixed Traffic** | **Autonomous Driving**

"HD and AD will coexist for several decades… It is still necessary to learn and model HD."

Motivation

- **Human Driving Modeling (HDM)**
  - from **data** to **policy** (world as it is): heterogeneity, uncertainty, …
- **Autonomous Driving Modeling (ADM)**
  - from **goal** to **policy** (world as it should be): design reward and loss …
- HDM is _descriptive_ and _generative_. ADM is _prescriptive_ and _normative_.

Human Driving (_descriptive_):
"How do humans drive?"



Autonomous Driving (_prescriptive_):
"How should an AV drive?"

[Wenshuo Wang, Letian Wang, **Chengyuan Zhang**, Changliu Liu, and Lijun Sun. "Social interactions for autonomous driving: A review and perspectives." Foundations and Trends® in Robotics 10, no. 3-4 (2022): 198-376.]

Motivation

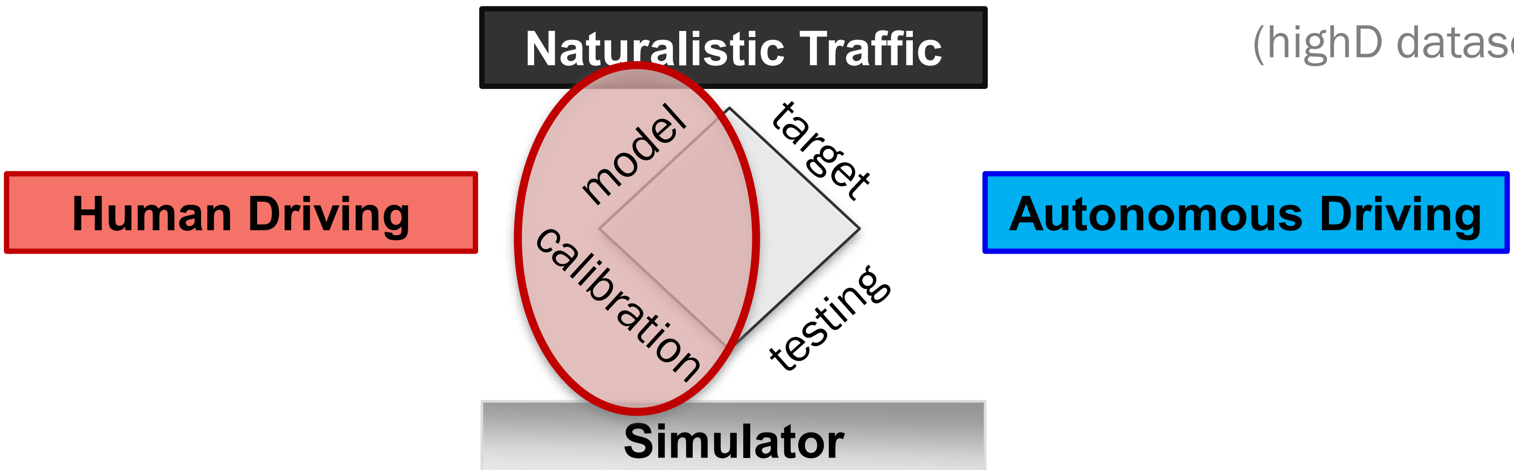
Aspect	HDM — world as it is	ADM — world as it should be
Concepts	Explains and generates human actions with memory and heterogeneity. Descriptive, stochastic policy $\pi_H(a \mid x_{0:t}, z)$	Under explicit objectives and constraints. Prescriptive, optimal policy $\pi_{AV}(x) = \arg\min_a J(x, a)$
Applications	Realistic human agents in simulators; traffic flow studies; policy impact via micro-to-macro scaling; scenario discovery	On-road autonomy; planning and control; safety envelopes; mission success
Evaluation metrics	Human-likeness, Social responsiveness, Flow realism	Safety, Comfort, Rule and right-of-way compliance, Risk margin
Role in sim	Makes the world believable	Succeeds within that world

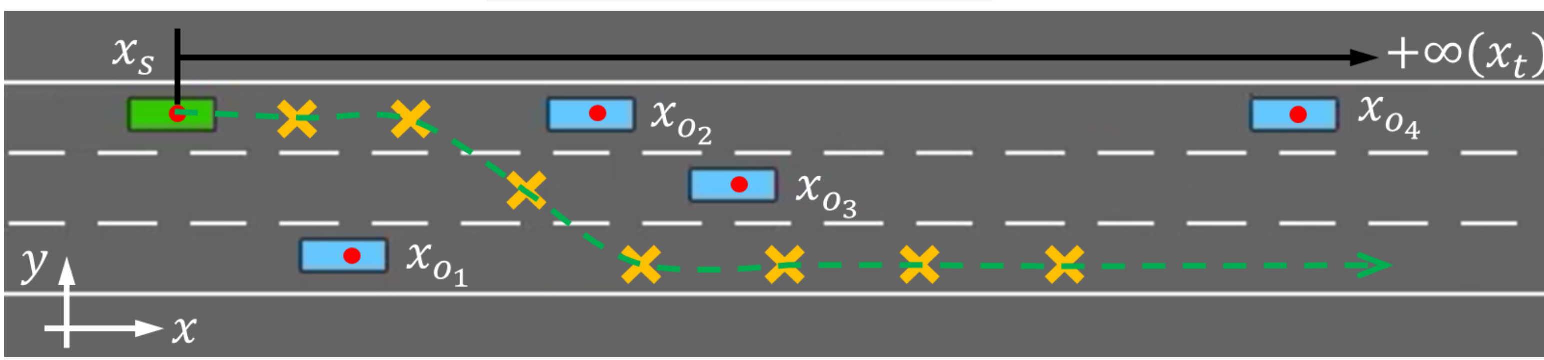
HDM is judged by *human-likeness* and *realism* of the world it creates. **ADM** is judged by *safety, rule compliance, comfort, and reliability* inside that world.

[Wenshuo Wang, Letian Wang, **Chengyuan Zhang**, Changliu Liu, and Lijun Sun. "Social interactions for autonomous driving: A review and perspectives." Foundations and Trends® in Robotics 10, no. 3-4 (2022): 198-376.]

Motivation

(highD dataset)

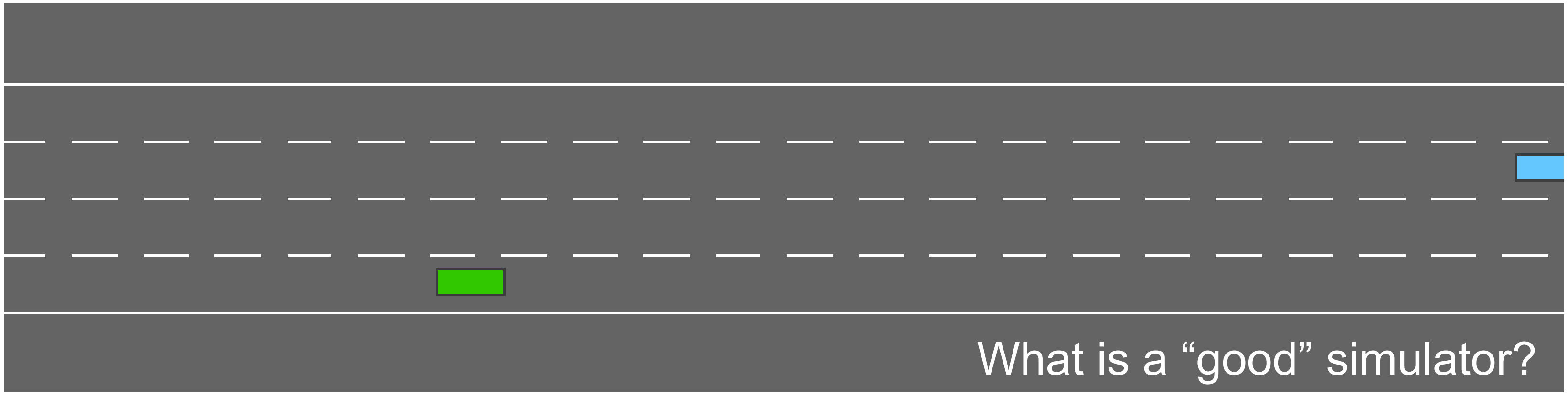




(a scene developed with highway-env)

Motivation

- The goal of traffic simulations:
  - Past: reproduce traffic phenomenon.
  - Future: support the development and test of Autonomous Driving:
    - **Reinforcement learning** for traffic control/management;
    - Human-in-the-loop **simulations**;
    - **Safety**, **predictability**, and **uncertainty**;



(a demo developed with highway-env)

A realistic traffic simulator is not just a convenience but a necessity!

How do we model the human driving behaviors?
How do we simulate human-like behaviors?

Outline

I. **Background and Problem Formulation**

II. **Modeling Continuous Uncertainty in Car-Following (CF) Behaviors**

W1. Bayesian Calibration of CF Models (CFMs) with Gaussian Processes

W2. Bayesian Dynamic Regression of CFMs with Autoregressive Errors

III. **Modeling Discrete Variability and Latent Structure in CF Behaviors**

W3. & W4. Latent Driving Pattern Modeling Using A Bayesian GMM

W5. Structured Driving Pattern Modeling Using Matrix Normal Mixture Model

W6. Regime Switching Models for Interpretable Behavioral Segmentation

IV. **Deep Probabilistic Models for Complex Driving Behavior**

W7. Neural Models with Structured Temporal Uncertainty

W8. Mapping the Subjective Risk Landscape of Continuous Human Action

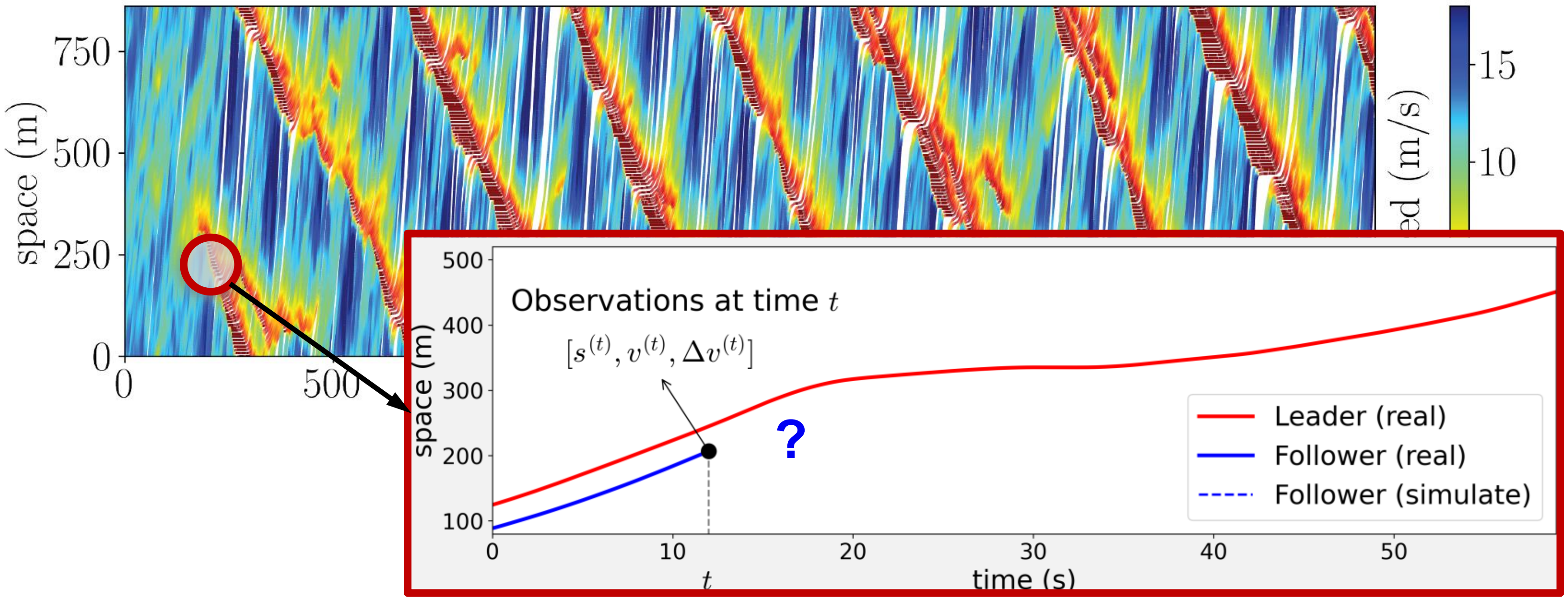
W9. Stochastic Calibration of CFMs via Simulation-Based Inference

V. **Discussion and Conclusions**

Outline

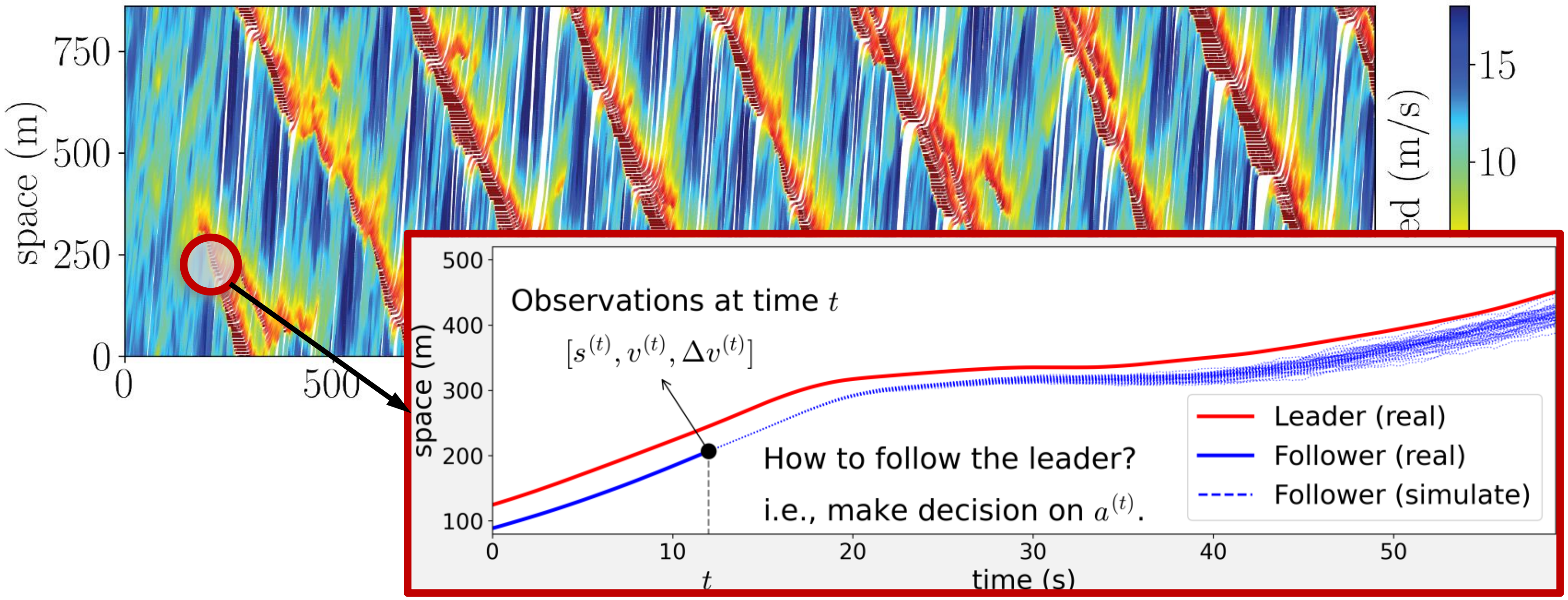
Background

- How would the vehicle react in response to the leading vehicle?



Background

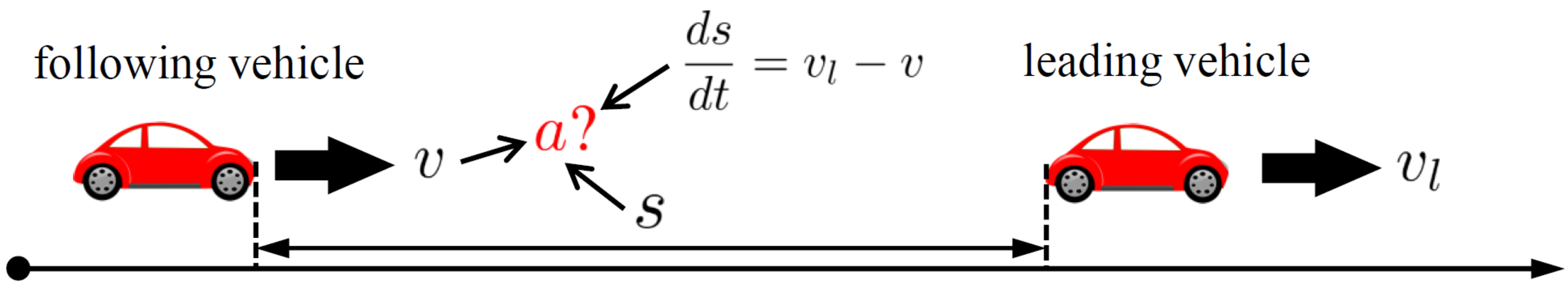
- How would the vehicle react in response to the leading vehicle?



How do we learn a good model?

"All models are wrong, but some are useful," by George Box

Intelligent driver model



- **Intelligent Driver Model (IDM)** (Treiber et al. 2000)

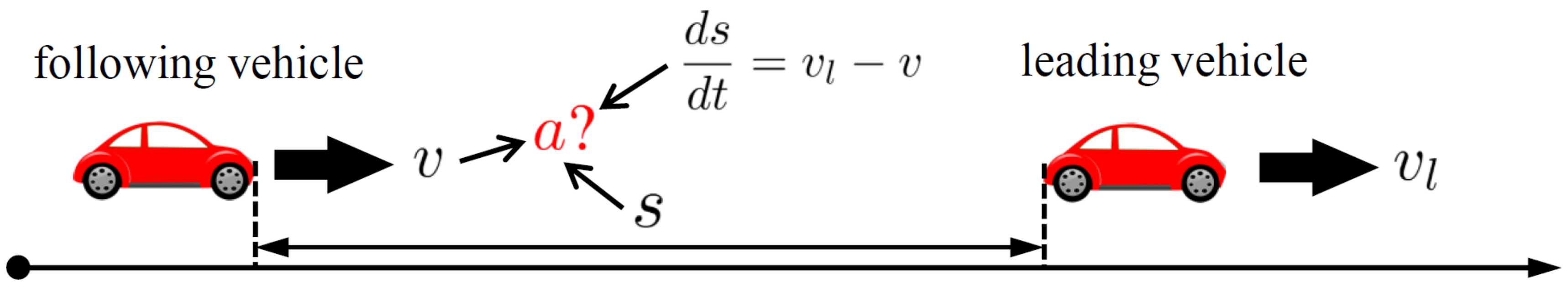
$$a_{\mathrm{IDM}} = \alpha \left(\underbrace{\boxed{1 - \left(\frac{v}{v_0}\right)^{\delta}}}_{\text{free-flow}} - \underbrace{\boxed{\left(\frac{s^*(v, \Delta v)}{s}\right)^2}}_{\text{interaction}} \right)$$

$$s^*(v, \Delta v) = s_0 + s_1 \sqrt{\frac{v}{v_0}} + v\,T + \frac{v\,\Delta v}{2\sqrt{\alpha\,\beta}}$$

- v_0: desired speed;
- s_0: jam spacing;
- T: time headway;
- α: maximum acceleration;
- β: comfortable deceleration rate.

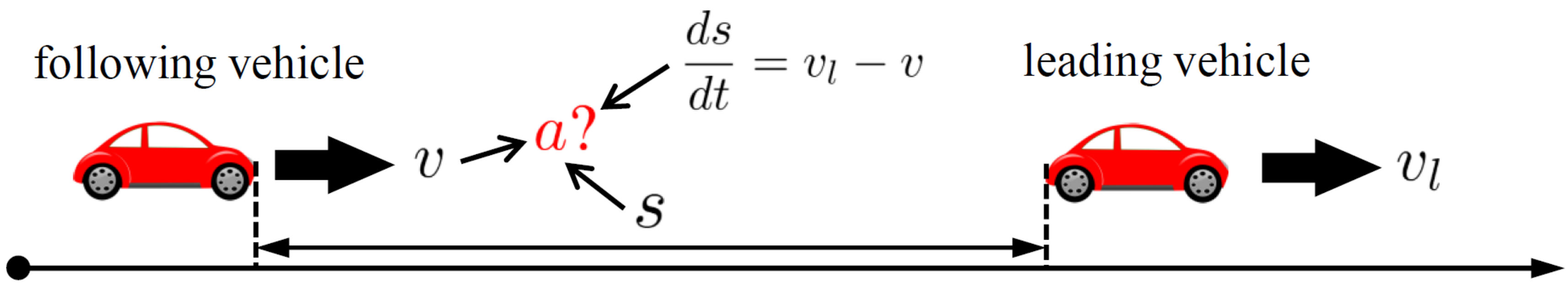
$$\boldsymbol{\theta} = \underbrace{[v_0, s_0, T, \alpha, \beta]}_{\text{parameter set}}$$

Intelligent driver model

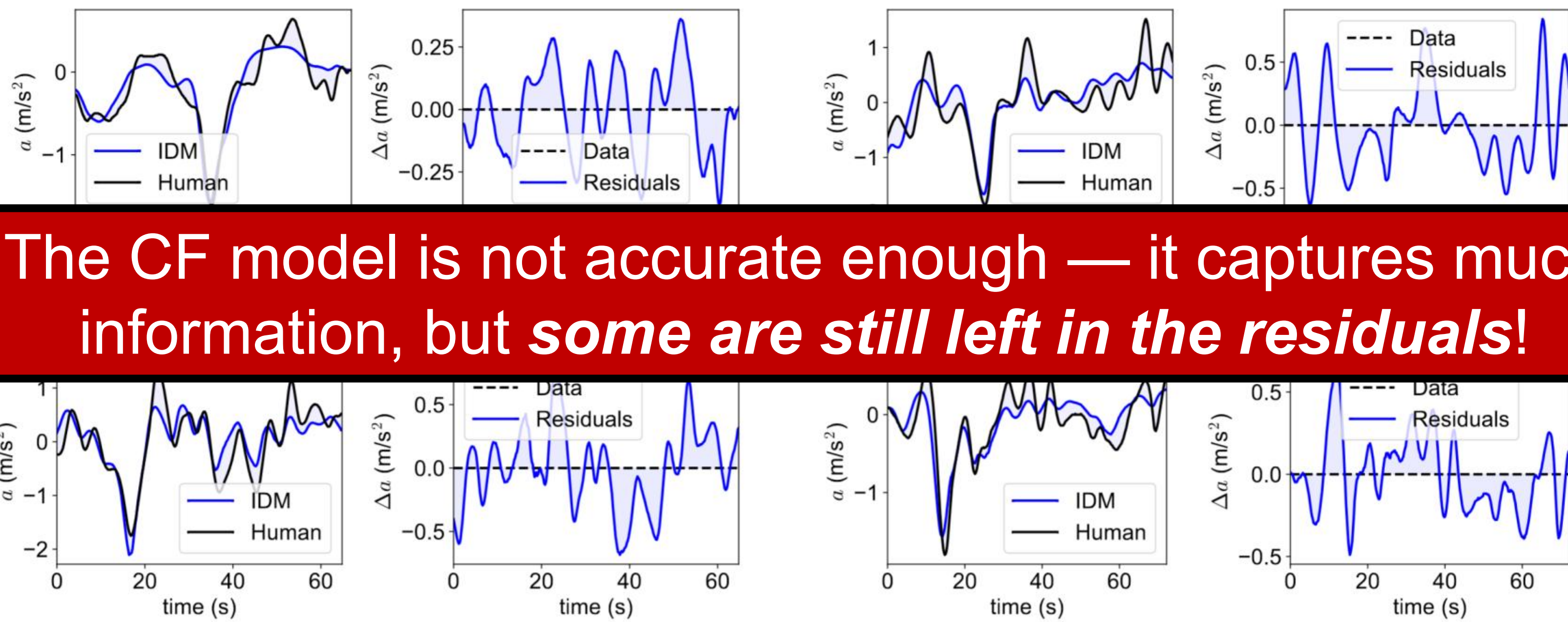


- IDM assumes: $a^{(t)} \approx a^{(t)}_{\text{IDM}}$. ➡ Likelihood: $\mathcal{N}(\hat{a}^{(t)} | a^{(t)}_{\text{IDM}}, \sigma^2_\epsilon)$
- Calibration by MLE: $\max_{\boldsymbol{\theta}} \prod_{t=1}^{T} \text{likelihood}$ and $\boldsymbol{\theta} = \underbrace{[v_0, s_0, T, \alpha, \beta]}_{\text{parameter set}}$
- Loss function: $\min_{\boldsymbol{\theta}} \frac{1}{T} \sum_{t=1}^{T} (a^{(t)}_{\text{IDM}} - \hat{a}^{(t)})^2$

Intelligent driver model



- IDM assumes: $a^{(t)} \approx a^{(t)}_{\text{IDM}}$. Let's visualize the residuals $a^{(t)} - a^{(t)}_{\text{IDM}}$.



The CF model is not accurate enough — it captures much information, but ***some are still left in the residuals***!

What is missing?

Our Targets:

- How do we model the human driving behaviors?
- How do we simulate human-like behaviors?

What are the characteristics of human driving behaviors?

- Memory and hysteresis;
- Heterogeneity;
- Stochasticity and uncertainty;
- Social interaction;
- Imperfect perception and delay;
- Driving regime switching;
- Adaptation and learning;
- ...

In this presentation, we will address all of these key characteristics with the two solutions

Problem: imperfect CFMs + unmodeled information (informative residuals)

- ✓ **Solution A**: explicitly model the residual process
- ✓ **Solution B**: build a better CFM by involving more information

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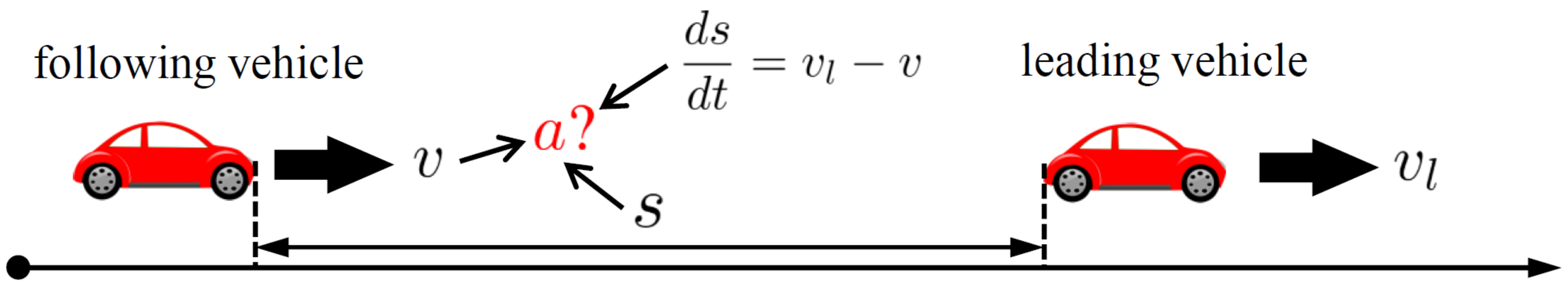
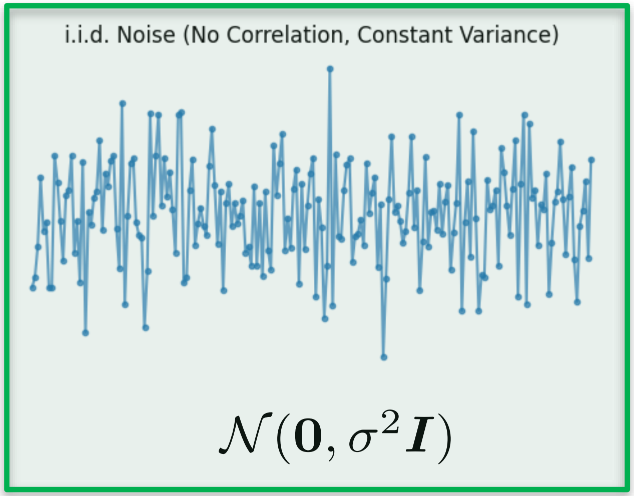
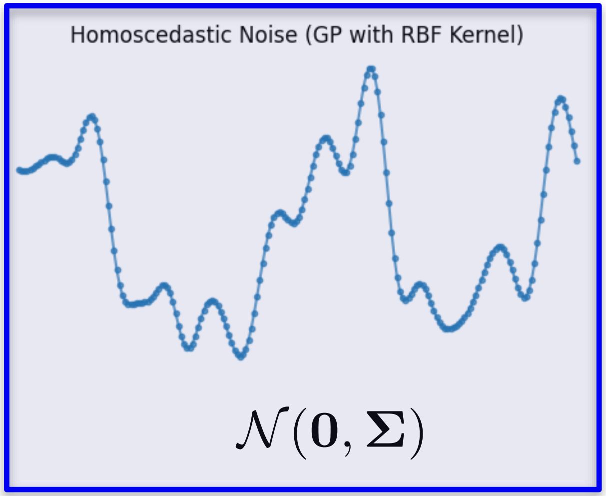
Temporal correlations in driving behaviors

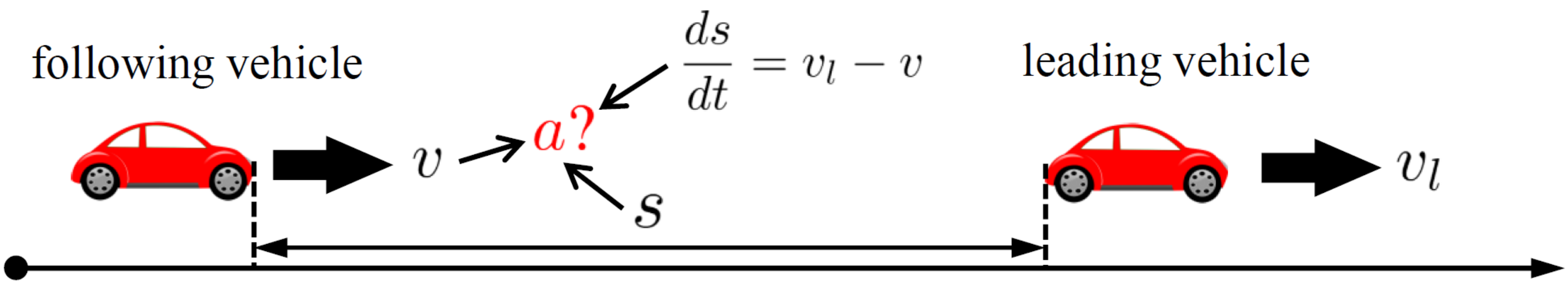


- **Solution A: explicitly model the residual process**
  - **Temporally correlated errors**





Temporal correlations in driving behaviors



- **Solution A: explicitly model the residual process**
  - **Temporally correlated errors**

Core contribution:

- Therefore, we assume

temporal correlations

(captures how past behaviors influence future behaviors)

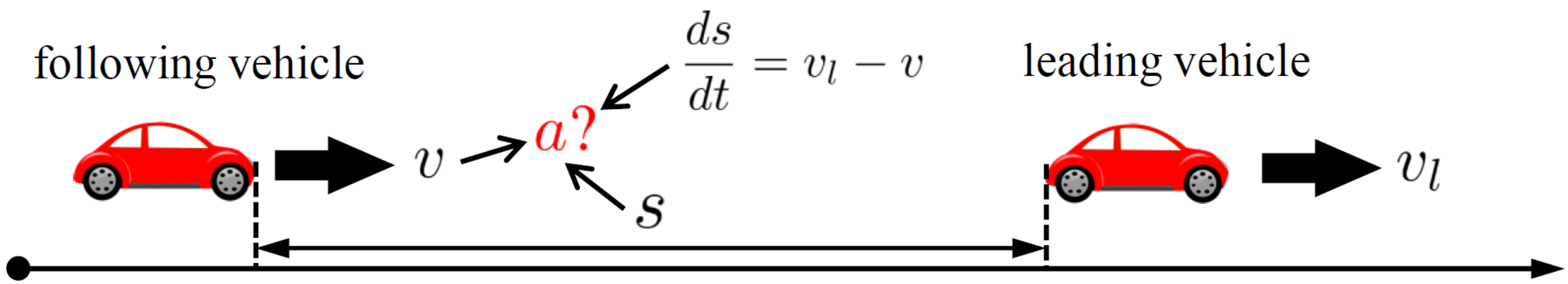
$$a(x,t) \approx \boxed{f_{\mathrm{CFM}}(x;\boldsymbol{\theta})} + \boxed{\delta(t)}$$

(Zhang et al. 2025)

mean car-following model

Inspired by GLS, see my post From Ordinary Least Squares (OLS) to Generalized Least Squares (GLS)

The general form of car-following models



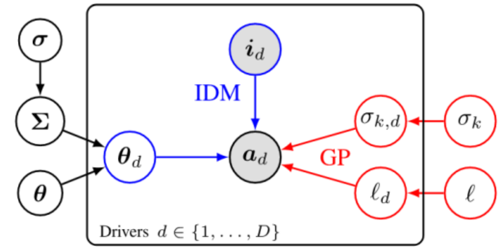
- We assume: $a(x,t) \approx \boxed{f_{\mathrm{CFM}}(x;\boldsymbol{\theta})} + \boxed{\delta(t)}$ (Zhang et al. 2025)
- IDM assumes: $a(x,t) \approx \boxed{a_{\mathrm{IDM}}(x;\boldsymbol{\theta})}$. (Treiber et al. 2000)

<u>Missed the temporal part</u> $\boxed{\delta(t)}$

TO-DO:

- Consider $\delta(t)$ in learning/calibration;
- Model $\delta(t+1)|\delta(t)$ in simulation.

Memory-Augmented IDM (MA-IDM) (Zhang and Sun 2024)



How to model $\delta(t)$ and $\delta(t+1)|\delta(t)$?

- We assume:

$$a(x,t) = \boxed{f_{\text{CFM}}(x;\boldsymbol{\theta})} + \boxed{\delta(t)} + \boxed{\epsilon_t}, \quad \epsilon_t \overset{i.i.d.}{\sim} \mathcal{N}(0,\sigma_\epsilon^2)$$

- Bayesian IDM assumes:

$$a^{(t)} = \boxed{a^{(t)}_{\text{IDM}}} + \boxed{\epsilon_t, \quad \epsilon_t \overset{i.i.d.}{\sim} \mathcal{N}(0,\sigma_\epsilon^2)} \qquad \Rightarrow \boldsymbol{a}|\boldsymbol{i},\boldsymbol{\theta} \sim \mathcal{N}(\boxed{\boldsymbol{a}_{\text{IDM}}}, \boxed{\sigma_\epsilon^2 \boldsymbol{I}})$$

incorrect

Vector form with Multivariate Normal

- MA-IDM assumes:

$$a^{(t)} = \boxed{a^{(t)}_{\text{IDM}}} + \boxed{a^{(t)}_{\text{GP}}} + \boxed{\epsilon_t} \qquad \Rightarrow \boldsymbol{a}|\boldsymbol{i},\boldsymbol{\theta} \sim \mathcal{N}(\boxed{\boldsymbol{a}_{\text{IDM}}}, \boxed{\boldsymbol{K}} + \boxed{\sigma_\epsilon^2 \boldsymbol{I}})$$

mean model residuals *i.i.d.* error

where ***K*** is a kernel matrix .

[Chengyuan Zhang and Lijun Sun. (2024). Bayesian calibration of the intelligent driver model. *IEEE Transactions on Intelligent Transportation Systems*.]

Memory-Augmented IDM (MA-IDM) (Zhang and Sun 2024)

- Bayesian IDM assumes:

$$a^{(t)} = a_{\mathrm{IDM}}^{(t)} + \epsilon_t,\ \ \epsilon_t \overset{i.i.d.}{\sim} \mathcal{N}(0, \sigma_\epsilon^2) \qquad \Rightarrow \boldsymbol{a} | \boldsymbol{i}, \boldsymbol{\theta} \sim \mathcal{N}(\boldsymbol{a}_{\mathrm{IDM}}, \sigma_\epsilon^2 \boldsymbol{I})$$

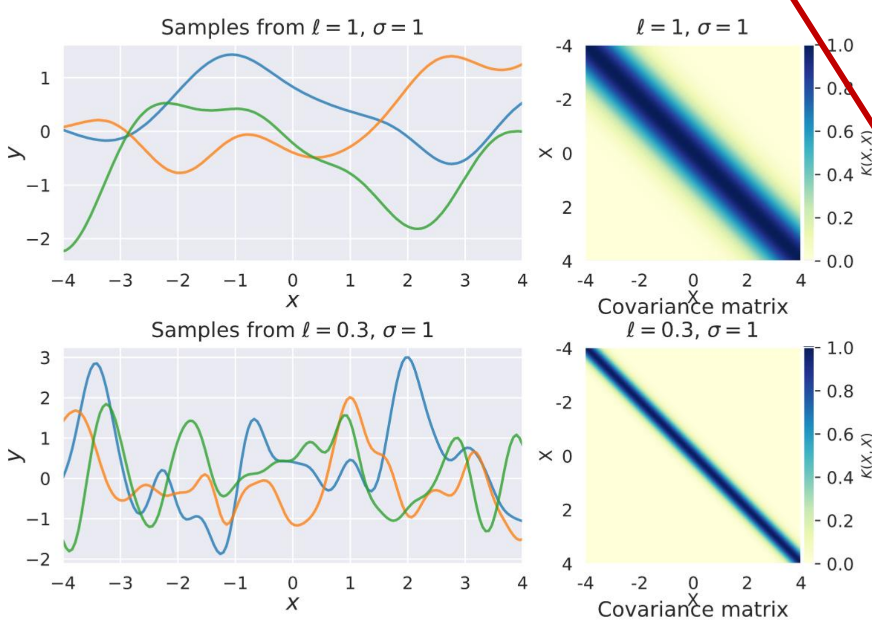
- MA-IDM assumes:

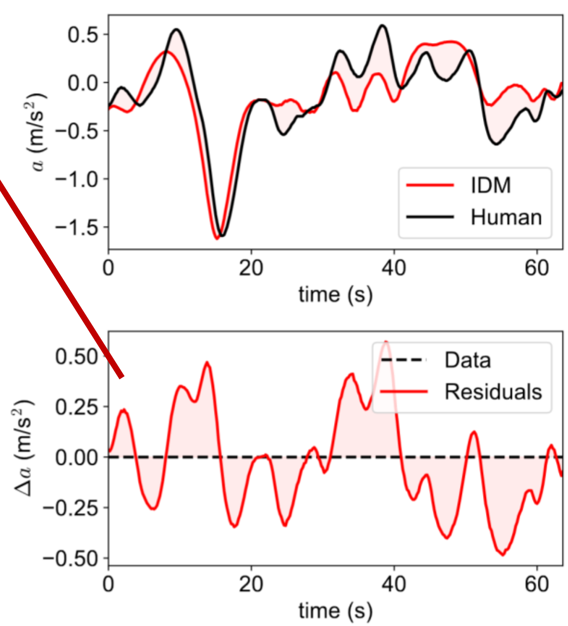
$$a^{(t)} = a_{\mathrm{IDM}}^{(t)} + \boxed{a_{\mathrm{GP}}^{(t)}} + \epsilon_t \qquad \Rightarrow \boldsymbol{a} | \boldsymbol{i}, \boldsymbol{\theta} \sim \mathcal{N}(\boldsymbol{a}_{\mathrm{IDM}}, \boxed{\boldsymbol{K}} + \sigma_\epsilon^2 \boldsymbol{I})$$

residuals

where $\boldsymbol{K}$ is a kernel matrix .

- Gaussian processes





Memory-Augmented IDM (MA-IDM) (Zhang and Sun 2024)

- Bayesian IDM assumes:

$$a^{(t)} = \boxed{a^{(t)}_{\mathrm{IDM}}} + \epsilon_t, \ \ \epsilon_t \overset{i.i.d.}{\sim} \mathcal{N}(0, \sigma_\epsilon^2) \qquad \Rightarrow \boldsymbol{a}|\boldsymbol{i}, \boldsymbol{\theta} \sim \mathcal{N}(\boldsymbol{a}_{\mathrm{IDM}}, \sigma_\epsilon^2 \boldsymbol{I})$$

- MA-IDM assumes:

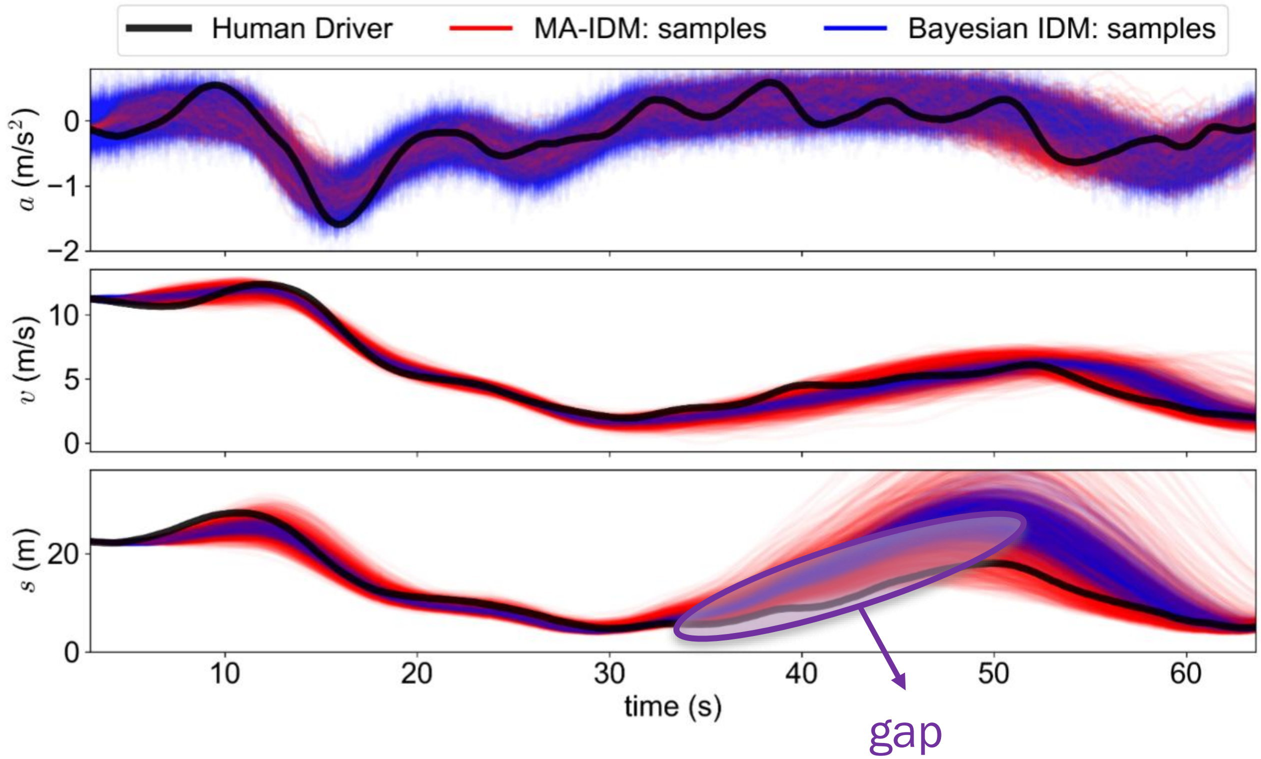
$$a^{(t)} = \boxed{a^{(t)}_{\mathrm{IDM}}} + \boxed{a^{(t)}_{\mathrm{GP}}} + \epsilon_t \qquad \Rightarrow \boldsymbol{a}|\boldsymbol{i}, \boldsymbol{\theta} \sim \mathcal{N}(\boldsymbol{a}_{\mathrm{IDM}}, \boldsymbol{K} + \sigma_\epsilon^2 \boldsymbol{I})$$

where ***K*** is a kernel matrix .

- Stochastic simulation for step t :

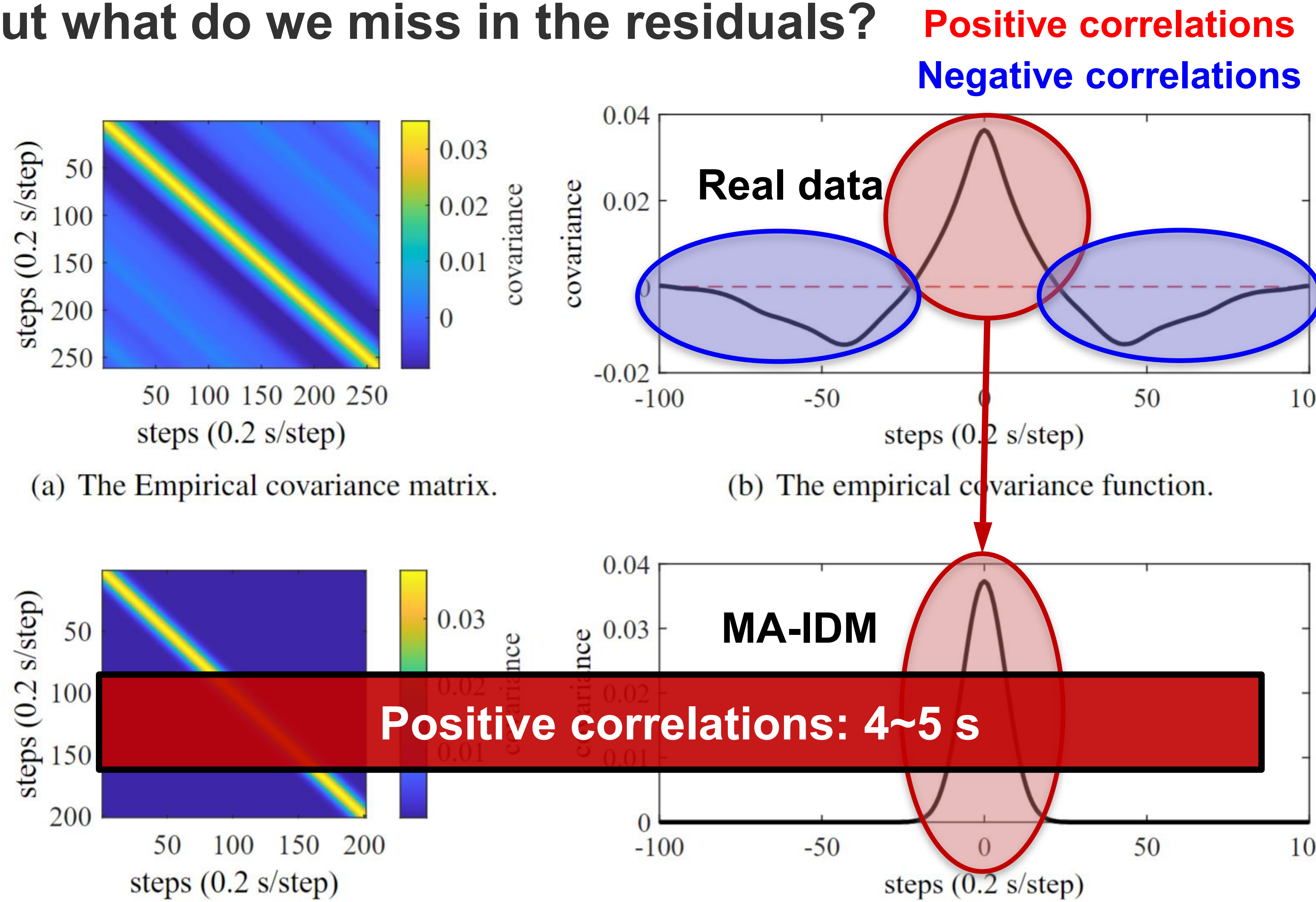
1) Obtain the first term $\boxed{a^{(t)}_{\mathrm{IDM},d}}$ by feeding $\boldsymbol{\theta}_d$ and inputs into the IDM function;
2) Draw a sample $\boxed{a^{(t)}_{\mathrm{GP},d} | \boldsymbol{a}^{(t-T:t-1)}_{\mathrm{GP},d}}$ at time t from the GP to obtain the temporally correlated information $a^{(t)}_{\mathrm{GP},d}$;

Simulations – Stochastic Simulation (MA-IDM v.s. B-IDM)



- MA-IDM has a better calibration result than B-IDM. Even B-IDM is with a large noise variance, it still cannot bridge the gap (*i.e.*, with bad uncertainty quantification.)

But what do we miss in the residuals?



(a) The Empirical covariance matrix.

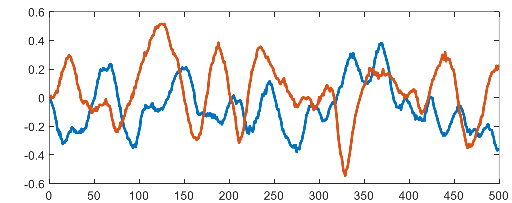
(b) The empirical covariance function.

(c) The RBF kernel matrix.

(d) The RBF kernel function.

Dynamic Regression Framework (Dynamic IDM)

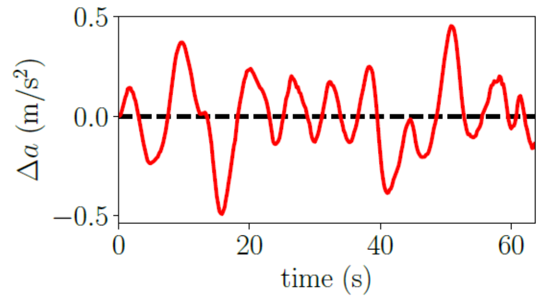
How to model $\delta(t)$ and $\delta(t+1)|\delta(t)$?



Two random series generated by AR(4)

- **We assume:**

$$a(x,t) = \boxed{a(x;\boldsymbol{\theta})} + \boxed{\delta(t)} + \boxed{\epsilon,} \; \epsilon \overset{i.i.d.}{\sim} \mathcal{N}(0,\sigma_\epsilon^2)$$



- **Dynamic IDM assumes:**

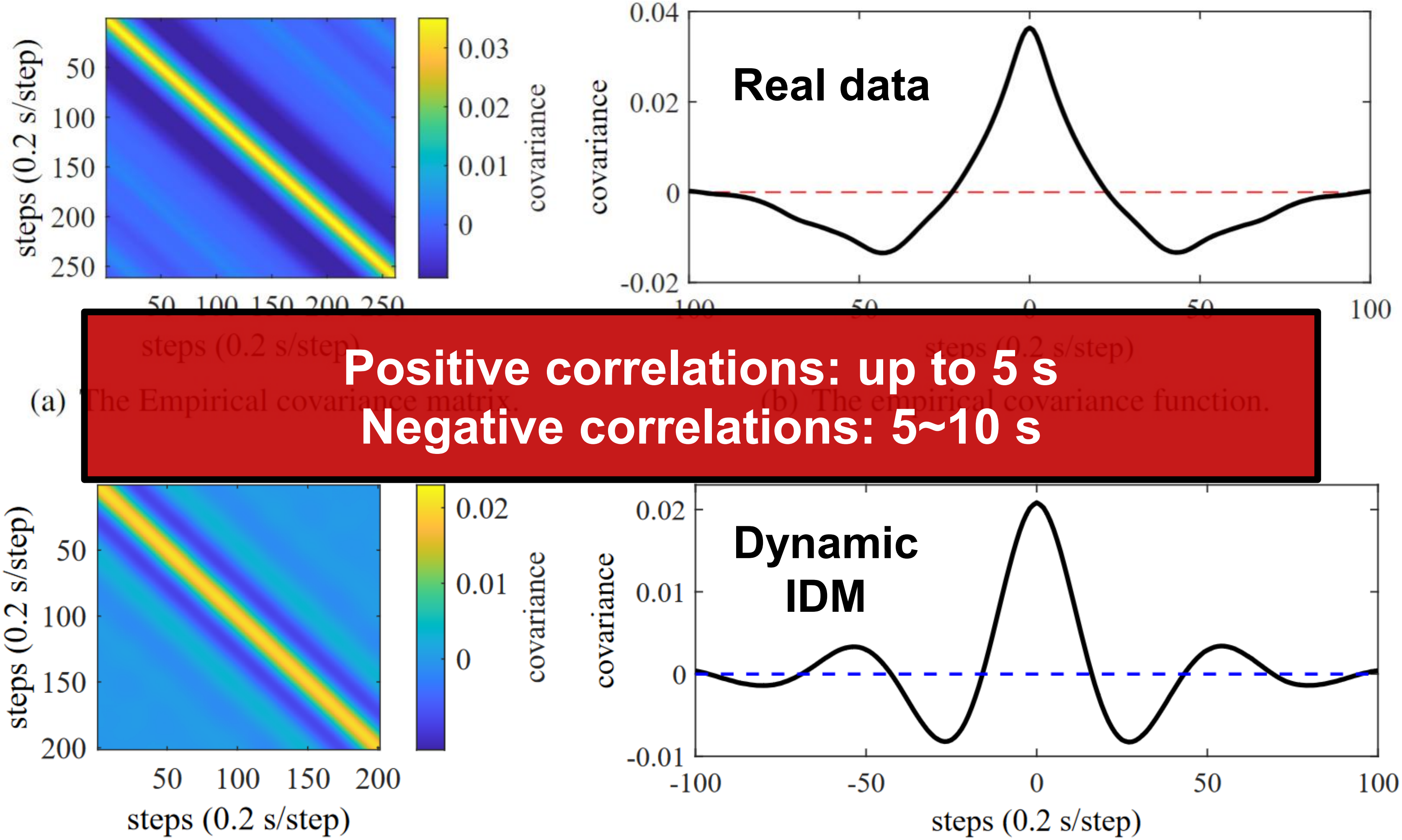
Autoregressive (AR) processes

$$a_d^{(t)} = \boxed{\mathrm{IDM}_d^{(t)}} + \varepsilon_d^{(t)},$$
$$\varepsilon_d^{(t)} = \boxed{\rho_{d,1}\varepsilon_d^{(t-1)} + \rho_{d,2}\varepsilon_d^{(t-2)} + \cdots + \rho_{d,p}\varepsilon_d^{(t-p)}} + \boxed{\eta_d^{(t)},}$$
$$\eta^{(t)} \overset{i.i.d.}{\sim} \mathcal{N}(0,\sigma_\eta^2).$$

ADVANTAGE: It involves rich information from **several historical steps** instead of using only one step.

[Chengyuan Zhang, Wenshuo Wang, and Lijun Sun. "Calibrating car-following models via Bayesian dynamic regression." (ISTTT25 Special Issue) Transportation Research Part C: Emerging Technologies 168 (2024): 104719.]

Experiments – Identified AR Parameters



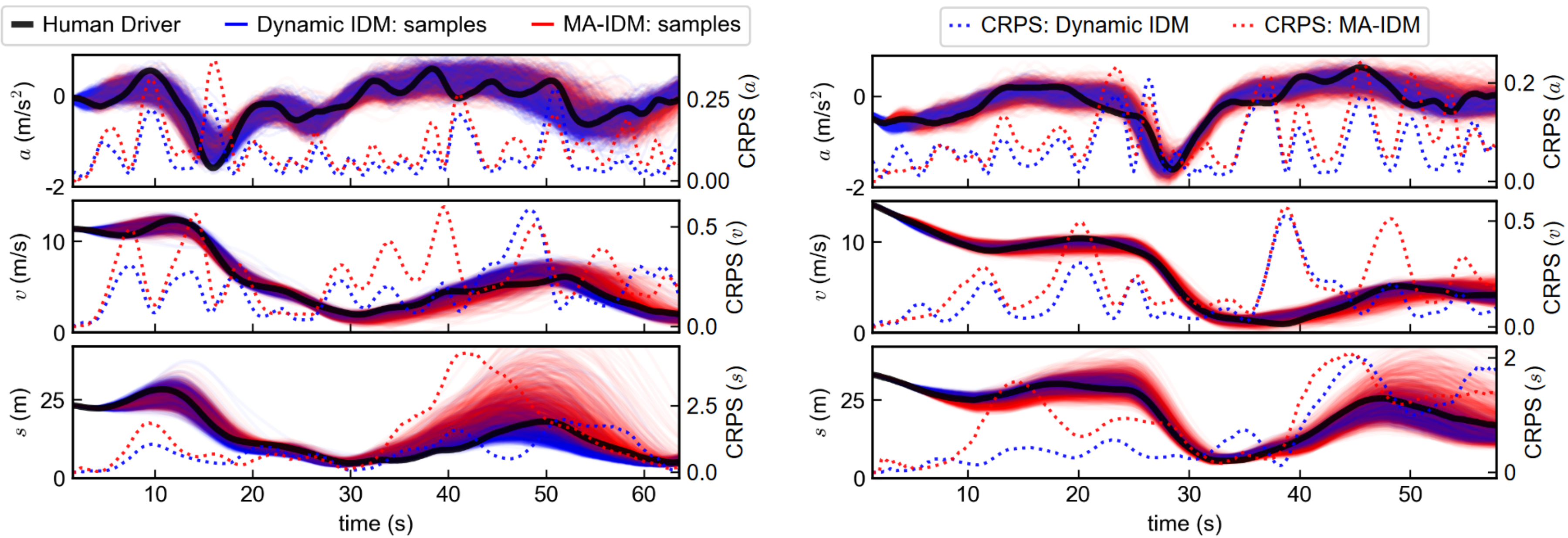
Positive correlations: up to 5 s
Negative correlations: 5~10 s

(e) The AR(5) covariance matrix.

(f) The AR(5) covariance functions.

$$\boldsymbol{\rho} = [0.874, 0.580, -0.105, -0.315, -0.071]$$

Stochastic Simulation (Dynamic IDM vs. MA-IDM)



(a) Truck driver #211. (b) Car driver #273.

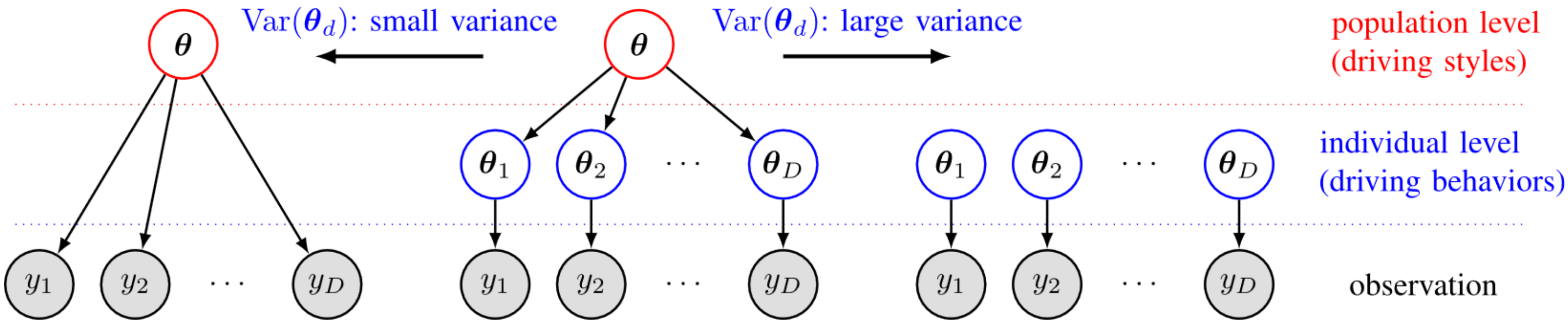
- Dynamic IDM has much lower variances than MA-IDM;

Dynamic IDM (AR+IDM) > MA-IDM (GP+IDM) >> Bayesian IDM > Traditional IDM

Lower variance unbiased probabilistic

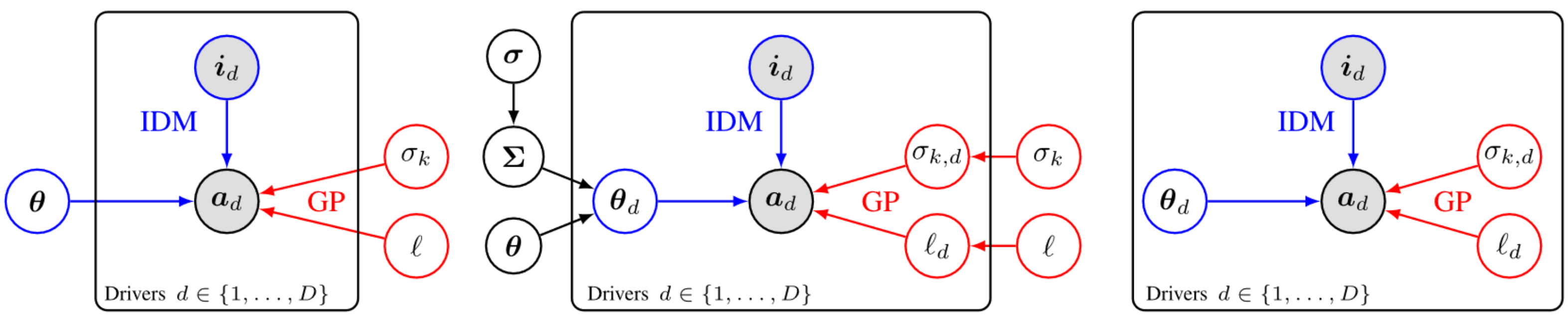
Modeling Driver Heterogeneity

- **Bayesian hierarchical model**



(a) Pooled model. (b) Hierarchical model. (c) Unpooled model.

(underfitting) **(overfitting)**



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W2. Bayesian Dynamic Regression o

Improved interpretability with Solution B

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Latent Variable Modeling

- **Solution B: build a better CFM by involving more information**

$$a(x,t) \approx f_{\mathrm{CF}}(x; \boldsymbol{\theta}_{z_t}), \ \ z_t \sim \begin{cases} \text{Gaussian Mixture} & \text{(W3-W5)} \\ \text{Markov Chain} & \text{(W6)} \end{cases}$$

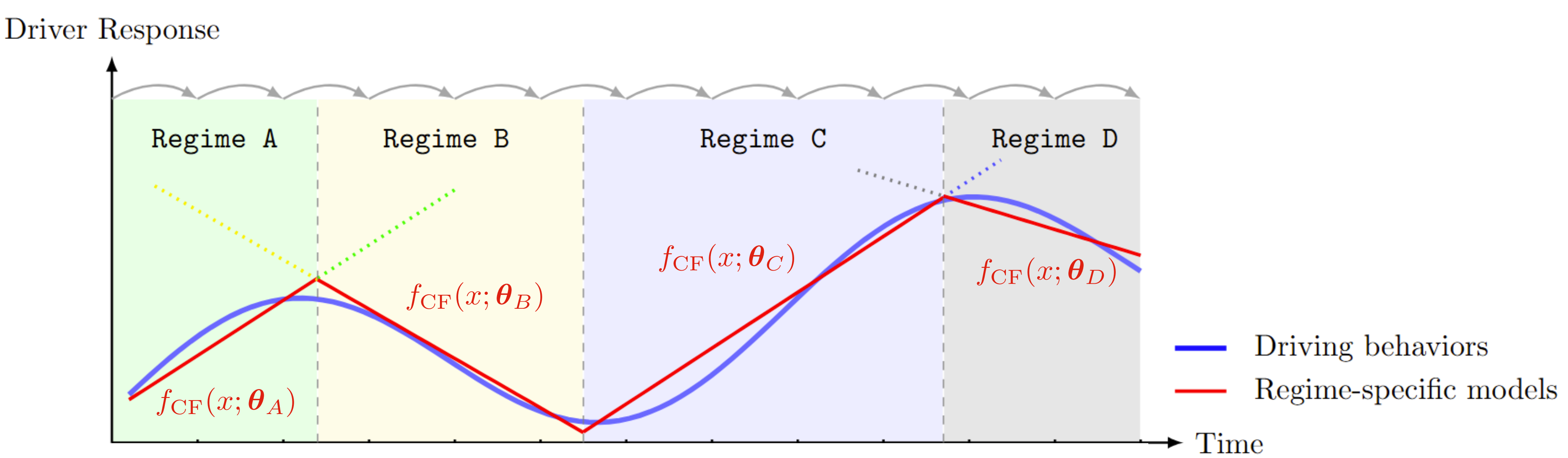
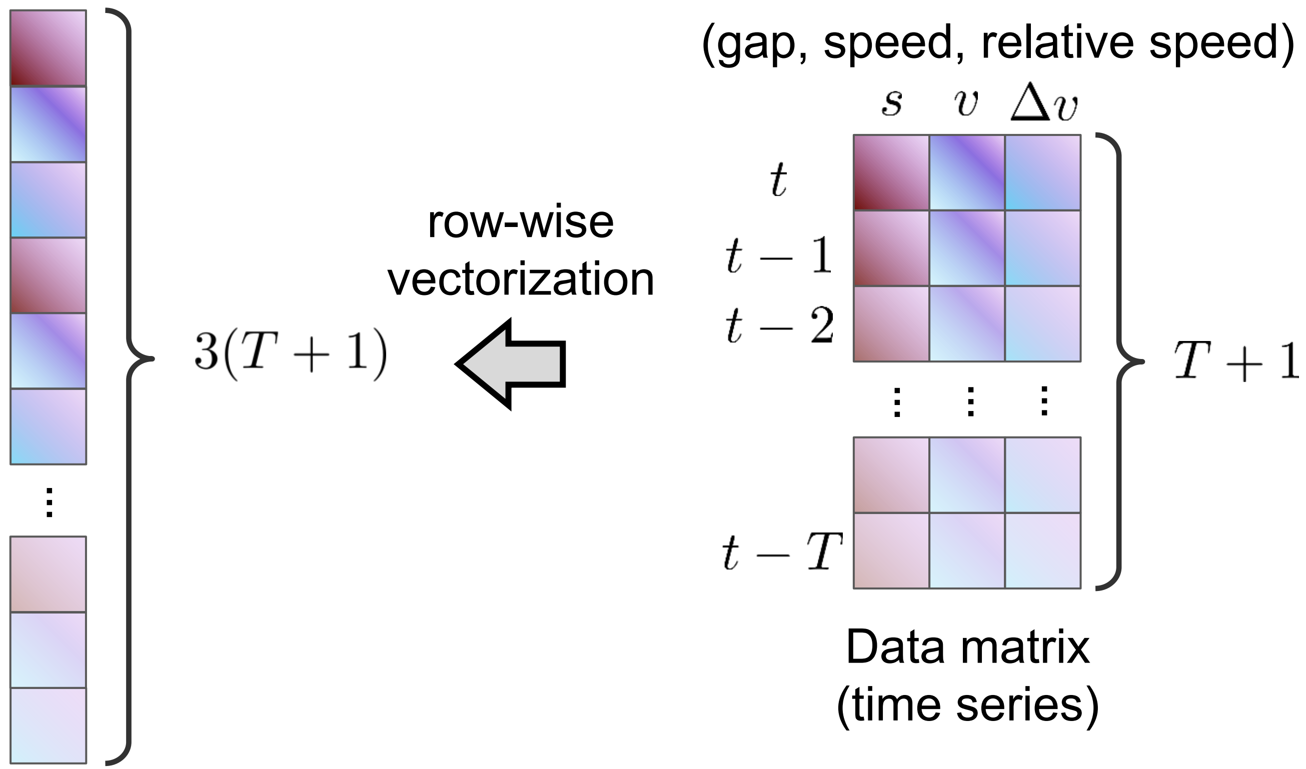


Figure: Conceptual illustration of the proposed framework.

Latent Variable Modeling – Gaussian Mixture Regression

$$a(x,t) \approx f_{\mathrm{CF}}(x; \boldsymbol{\theta}_{z_t}), \quad z_t \sim \text{Gaussian Mixture}$$

- IDM: parsimonious model without memory (historical information).



delay embedding
(vector)

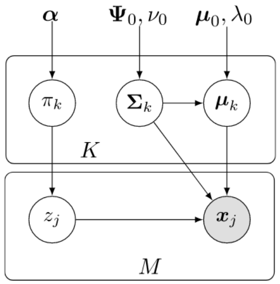
Data matrix
(time series)

providing historical information

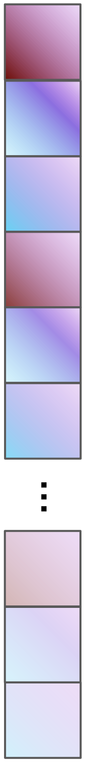
[Xiaoxu Chen, Chengyuan Zhang, Zhanhong Cheng, Yuang Hou, and Lijun Sun. "A bayesian gaussian mixture model for probabilistic modeling of car-following behaviors." IEEE Transactions on Intelligent Transportation Systems 25, no. 6 (2023): 5880-5891]

Latent Variable Modeling – Gaussian Mixture Regression

$$a(x,t) \approx f_{\mathrm{CF}}(x;\boldsymbol{\theta}_{z_t}), \quad z_t \sim \text{Gaussian Mixture}$$



- Gaussian Mixture Model (GMM)



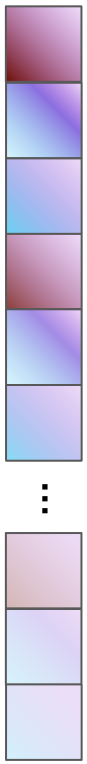
delay embedding
(vector)

$$\sim \sum_k \pi_k \mathcal{N}\left(\underbrace{\begin{bmatrix} \boldsymbol{\mu}_s^{(t)} \\ \boldsymbol{\mu}_v^{(t)} \\ \boldsymbol{\mu}_{\Delta v}^{(t)} \\ \vdots \\ \boldsymbol{\mu}_s^{(t-T)} \\ \boldsymbol{\mu}_v^{(t-T)} \\ \boldsymbol{\mu}_{\Delta v}^{(t-T)} \end{bmatrix}}_{\boldsymbol{\mu}_k \in \mathbb{R}^{3(T+1)}}, \underbrace{\begin{bmatrix} \ddots & & \\ & & \\ & & \ddots \end{bmatrix}}_{\boldsymbol{\Sigma}_k \in \mathbb{R}^{3(T+1)\times 3(T+1)}} \right)$$

Latent Variable Modeling – Gaussian Mixture Regression

$$a(x,t) \approx f_{\mathrm{CF}}(x;\boldsymbol{\theta}_{z_t}), \quad z_t \sim \text{Gaussian Mixture}$$

- Gaussian Mixture Regression (GMR) Model



delay embedding (vector)

$$\begin{bmatrix} \boldsymbol{\mu}_s^{(t)} \\ \boldsymbol{\mu}_v^{(t)} \\ \boldsymbol{\mu}_{\Delta v}^{(t)} \end{bmatrix} \Bigg| \begin{bmatrix} \boldsymbol{\mu}_s^{(t-1)} \\ \boldsymbol{\mu}_v^{(t-1)} \\ \boldsymbol{\mu}_{\Delta v}^{(t-1)} \\ \vdots \\ \boldsymbol{\mu}_s^{(t-T)} \\ \boldsymbol{\mu}_v^{(t-T)} \\ \boldsymbol{\mu}_{\Delta v}^{(t-T)} \end{bmatrix} \sim \sum_k \omega_k \mathcal{N}\left(\boldsymbol{\mu}_{t|t-1:t-T}, \boldsymbol{\Sigma}_{t|t-1:t-T}\right)$$

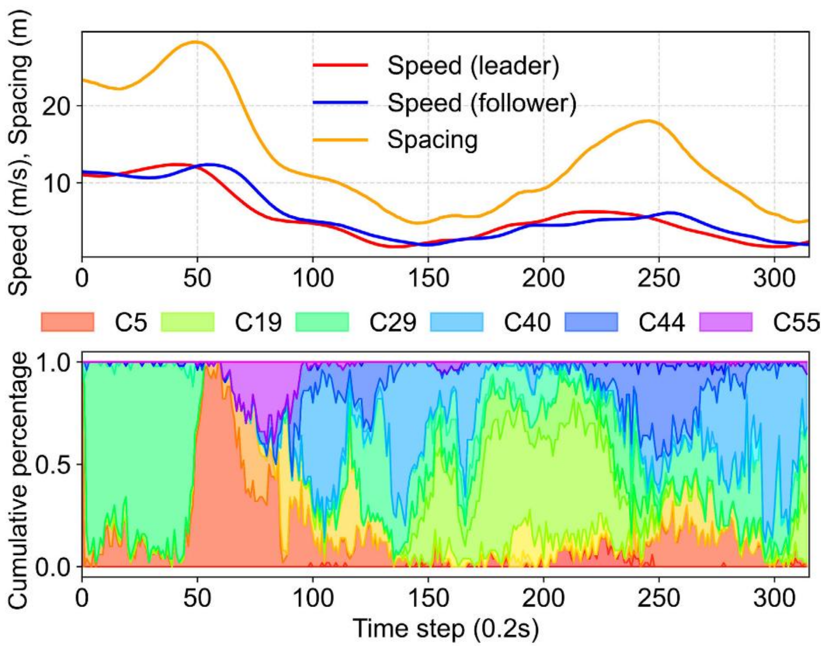
conditional Gaussian distribution

Then we can do simulations!

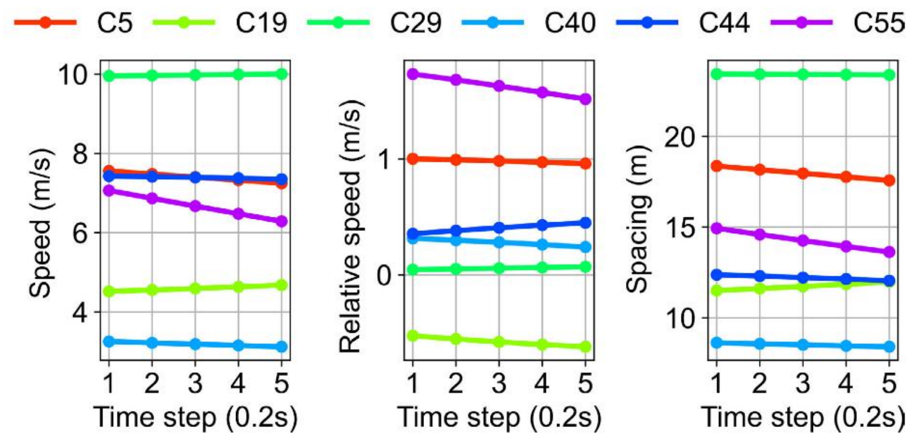
Latent Variable Modeling – Gaussian Mixture Regression

$$a(x,t) \approx f_{\mathrm{CF}}(x;\boldsymbol{\theta}_{z_t}), \quad z_t \sim \text{Gaussian Mixture}$$

- Gaussian Mixture Model (GMM)



5 example components

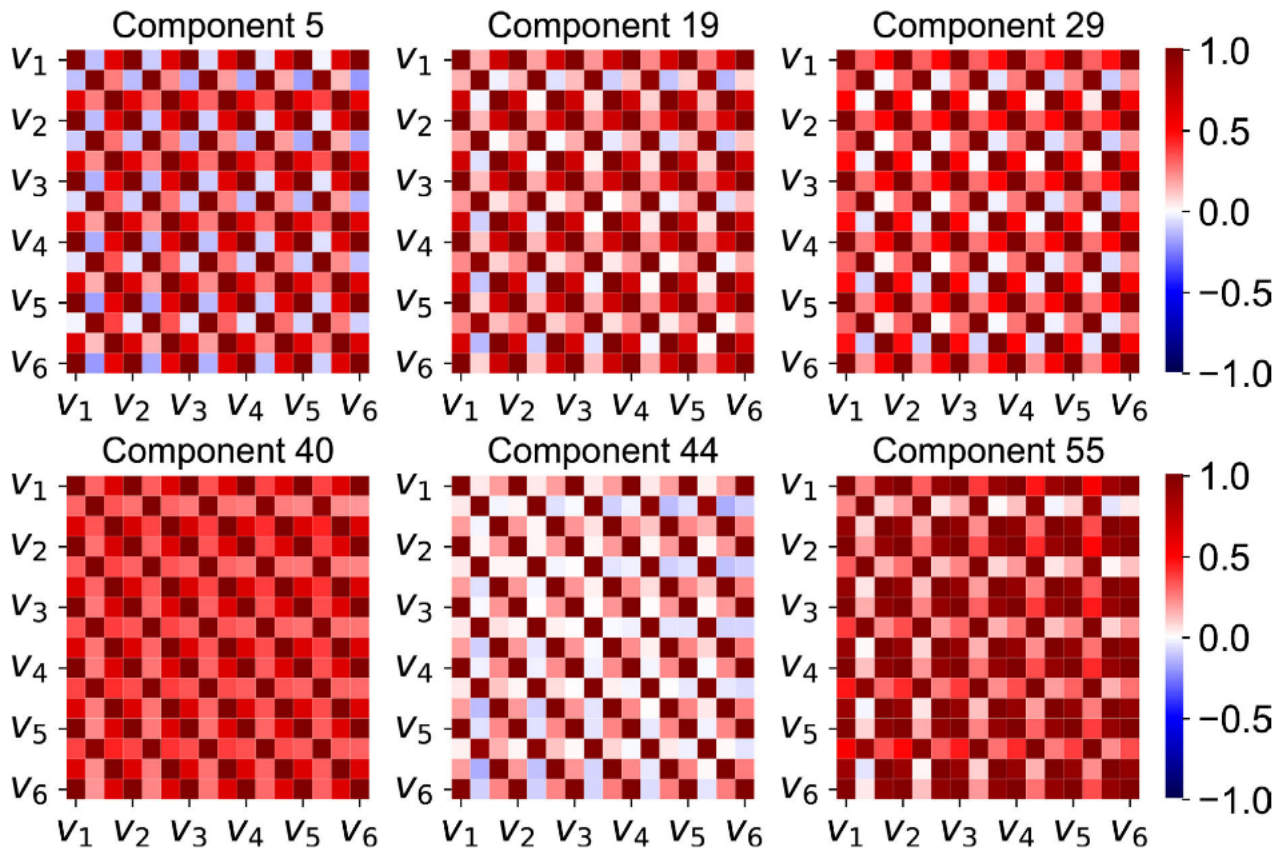


Free flow; gap closing; cruising; …

Latent Variable Modeling – Gaussian Mixture Regression

$$a(x,t) \approx f_{\mathrm{CF}}(x;\boldsymbol{\theta}_{z_t}), \quad z_t \sim \text{Gaussian Mixture}$$

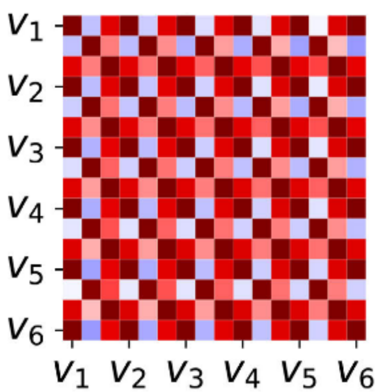
- Gaussian Mixture Model (GMM)



Latent Variable Modeling – Matrix Normal Mixture

$$a(x,t) \approx f_{\mathrm{CF}}(x;\boldsymbol{\theta}_{z_t}), \quad z_t \sim \text{Matrix Normal Mixture}$$

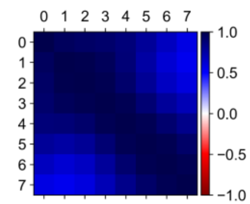
- Can we find a more efficient way to represent the big covariance matrix?



$\boldsymbol{\Sigma}_k \in \mathbb{R}^{3(T+1)\times 3(T+1)}$

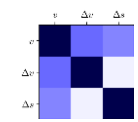
Overparameterization!

Matrix decomposition



$\boldsymbol{V}_k \in \mathbb{R}^{(T+1)\times(T+1)}$

$\otimes$



$\boldsymbol{U}_k \in \mathbb{R}^{3\times 3}$

Kronecker product

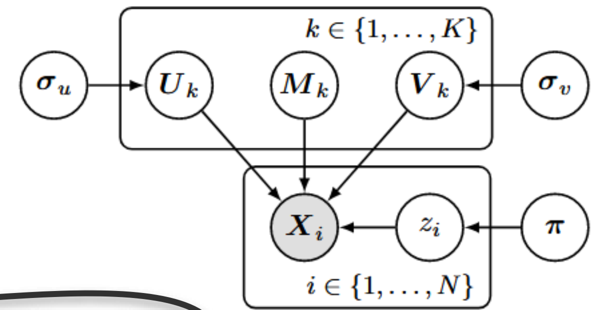
$$\boldsymbol{V} \otimes \boldsymbol{U} = \begin{bmatrix} v_{11}\boldsymbol{U} & \cdots & v_{1\tau}\boldsymbol{U} \\ \vdots & \ddots & \vdots \\ v_{\tau 1}\boldsymbol{U} & \cdots & v_{\tau\tau}\boldsymbol{U} \end{bmatrix}$$

[Chengyuan Zhang, Kehua Chen, Meixin Zhu, Hai Yang, and Lijun Sun. "Learning car-following behaviors using bayesian matrix normal mixture regression." In 2024 IEEE Intelligent Vehicles Symposium (IV), pp. 608-613. IEEE, 2024]

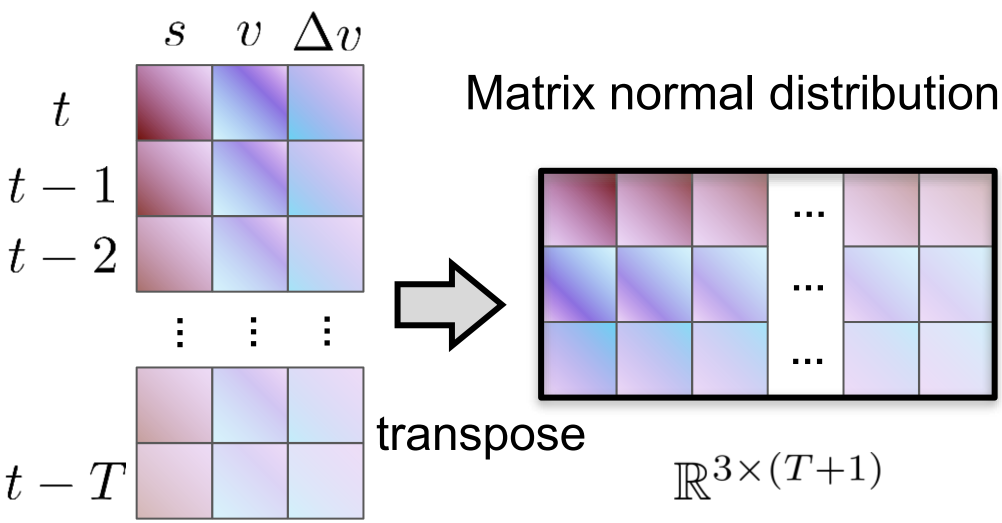
Latent Variable Modeling – Matrix Normal Mixture

$$a(x,t) \approx f_{\mathrm{CF}}(x;\boldsymbol{\theta}_{z_t}),\quad z_t \sim \text{Matrix Normal Mixture}$$

- Matrix Normal Mixture Model (MNMM)



(gap, speed, relative speed)



Data matrix
(time series)

Matrix normal distribution

$\mathbb{R}^{3\times(T+1)}$

$$\sim \sum_k \pi_k \mathcal{MN}\left(\boldsymbol{M}_k, \boldsymbol{U}_k, \boldsymbol{V}_k\right)$$

Mean matrix: $\boldsymbol{M}_k \in \mathbb{R}^{3\times(T+1)}$

Covariance matrices: $\begin{cases} \boldsymbol{U}_k \in \mathbb{R}^{3\times 3} \\ \boldsymbol{V}_k \in \mathbb{R}^{(T+1)\times(T+1)} \end{cases}$

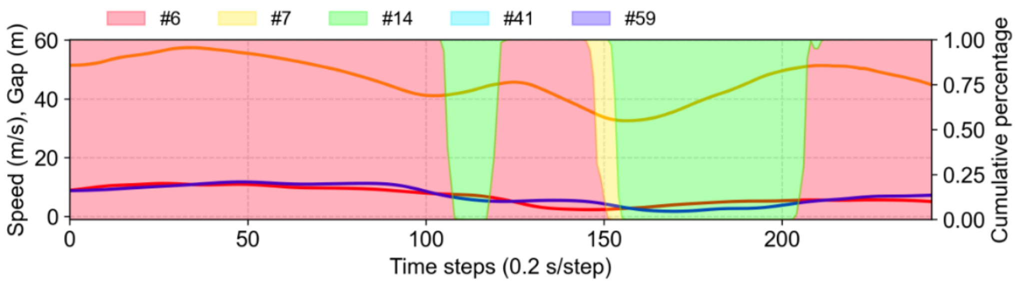
$$\boldsymbol{\Sigma}_k = \boldsymbol{V}_k \otimes \boldsymbol{U}_k$$

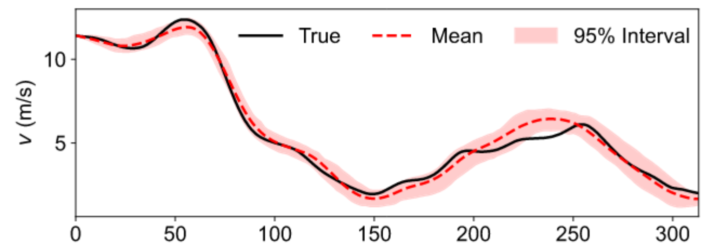
Latent Variable Modeling – Matrix Normal Mixture

$$a(x,t) \approx f_{\mathrm{CF}}(x;\boldsymbol{\theta}_{z_t}), \quad z_t \sim \text{Matrix Normal Mixture}$$

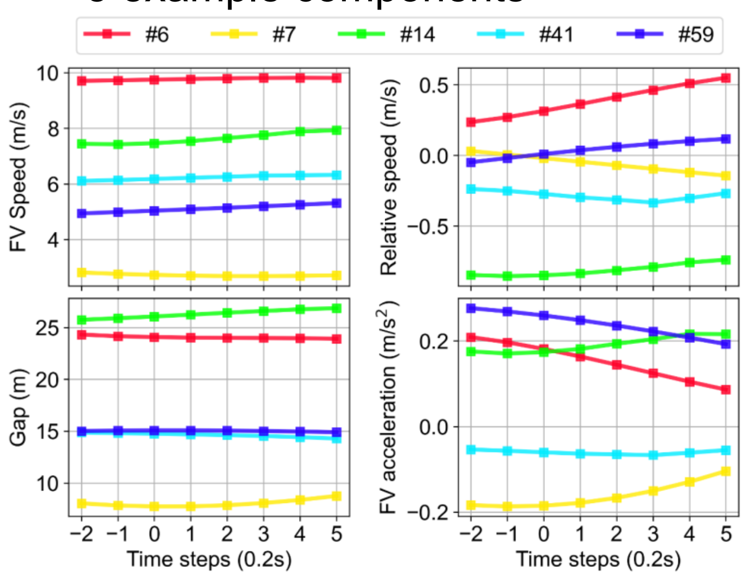
✓ Interpretability

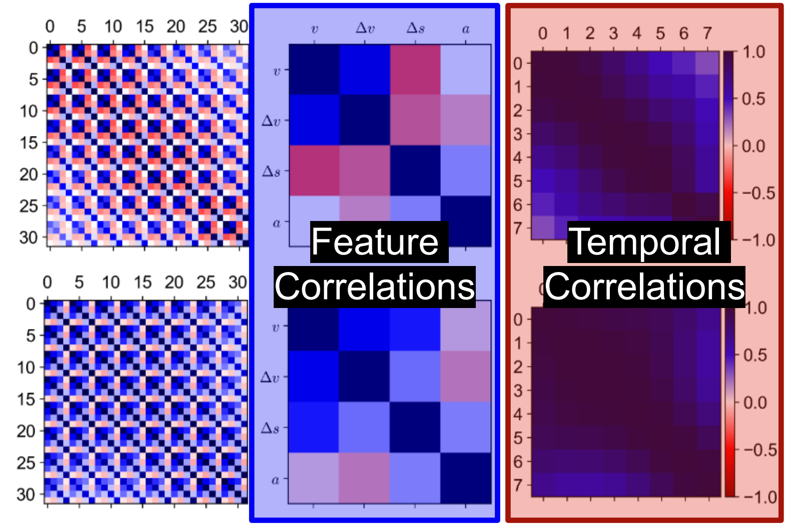
- Matrix Normal Mixture Regression (MNMR) Model





5 example components





Latent Variable Modeling – Quantify Interaction Intensity

$$a(x,t) \approx f_{\mathrm{CF}}(x;\boldsymbol{\theta}_{z_t}), \quad z_t \sim \text{Gaussian Mixture}$$

- **What is social interaction?**
  - A Quantifiable Definition: "A dynamic sequence of acts that mutually consider the actions and reactions of individuals through an information exchange process between two or more agents to maximize benefits and minimize costs."

[Wenshuo Wang, Letian Wang, Chengyuan Zhang, Changliu Liu, and Lijun Sun. "Social interactions for autonomous driving: A review and perspectives." Foundations and Trends® in Robotics 10, no. 3-4 (2022): 198-376.]

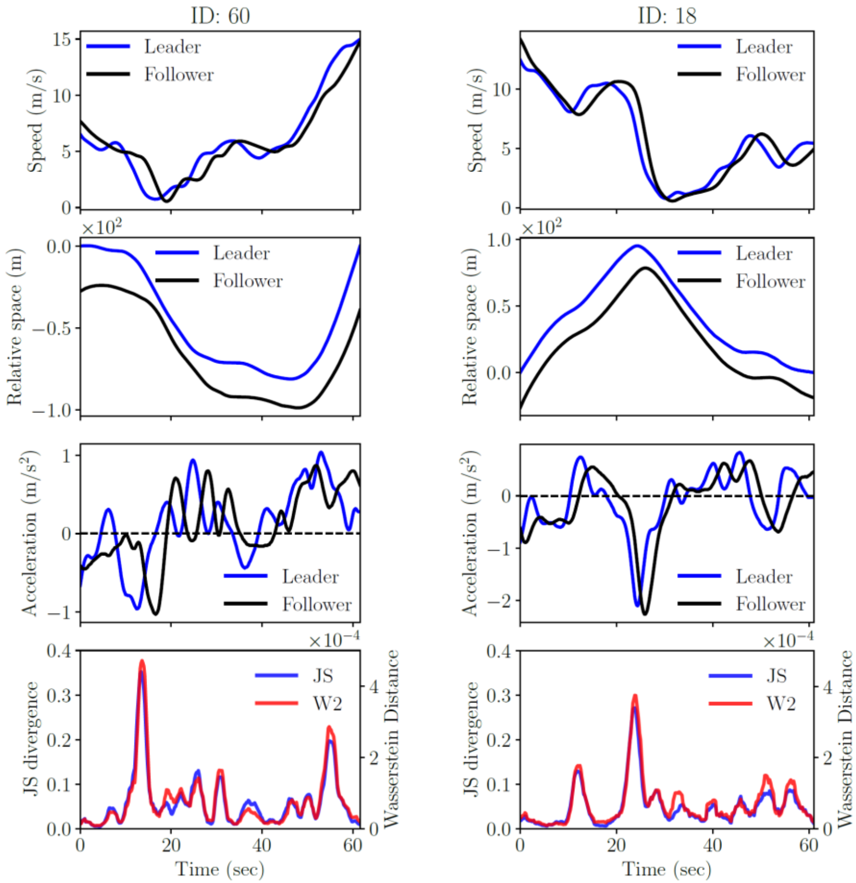
- **How to quantify interaction intensity?**
  - measure to what extent the conditional probability distribution shifts upon the leader's actions.

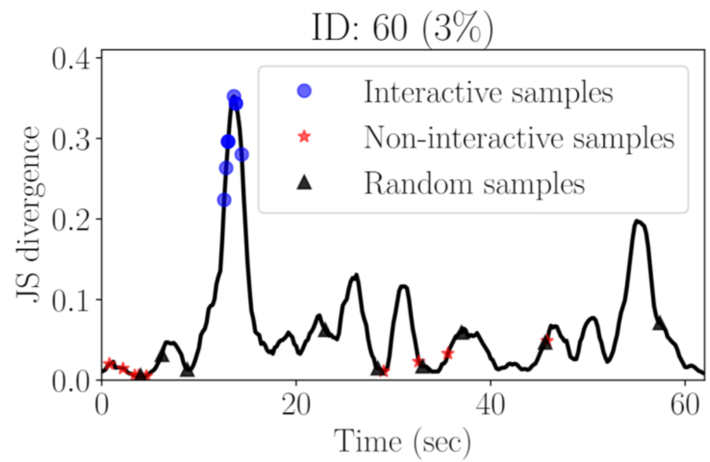
$$\mathcal{I}(\boldsymbol{a}_{\mathrm{foll}}, \boldsymbol{s}) := \mathcal{D}\Big(\underbrace{p(\hat{\boldsymbol{a}}_{\mathrm{foll}} | \boldsymbol{s}_{\mathrm{foll}}, \boldsymbol{s}_{\mathrm{lead}}, *)}_{\text{conditional dist. } f} \,\Big|\Big|\, \underbrace{p(\hat{\boldsymbol{a}}_{\mathrm{foll}} | \boldsymbol{s}_{\mathrm{foll}}, *)}_{\text{marginal dist. } g} \Big),$$

[Chengyuan Zhang, Rui Chen, Jiacheng Zhu, Wenshuo Wang, Changliu Liu, and Lijun Sun. "Interactive car-following: Matters but not always." In 2023 IEEE 26th International Conference on Intelligent Transportation Systems (ITSC), pp. 5120-5125. IEEE, 2023]

Latent Variable Modeling – Quantify Interaction Intensity

$$\mathcal{I}(\boldsymbol{a}_{\text{foll}}, \boldsymbol{s}) \coloneqq \mathcal{D}\Big(\underbrace{p(\hat{\boldsymbol{a}}_{\text{foll}} | \boldsymbol{s}_{\text{foll}}, \boldsymbol{s}_{\text{lead}}, *)}_{\text{conditional dist. } f} \Big|\Big| \underbrace{p(\hat{\boldsymbol{a}}_{\text{foll}} | \boldsymbol{s}_{\text{foll}}, *)}_{\text{marginal dist. } g}\Big),$$





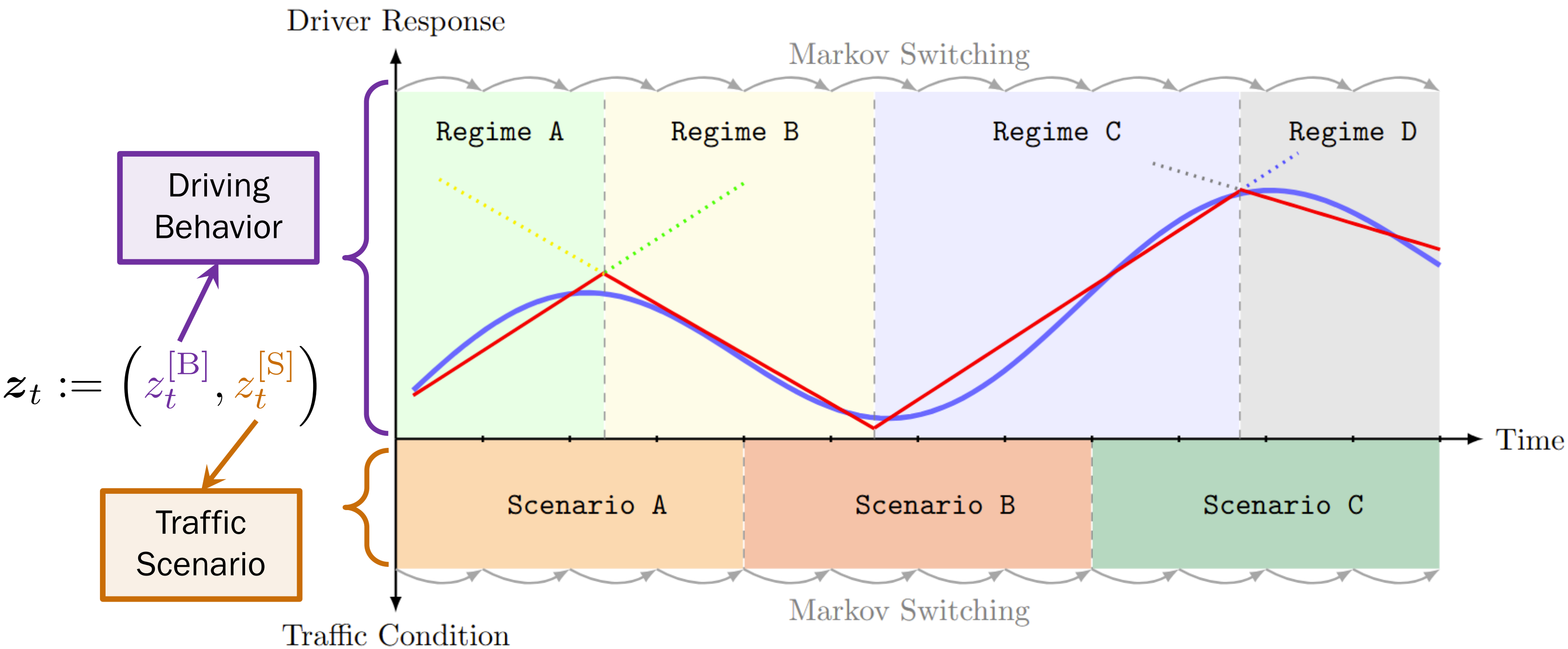
We can evaluate the interaction intensity and sample **interactive** and **non-interactive** cases!

(application: safety-critical scenario generation)

Latent Variable Modeling – Hidden Markov Model

$$a(x,t) \approx f_{\mathrm{CF}}(x;\boldsymbol{\theta}_{z_t}), \quad z_t \sim \text{Markov Chain}$$

- Factorial-HMM (FHMM)

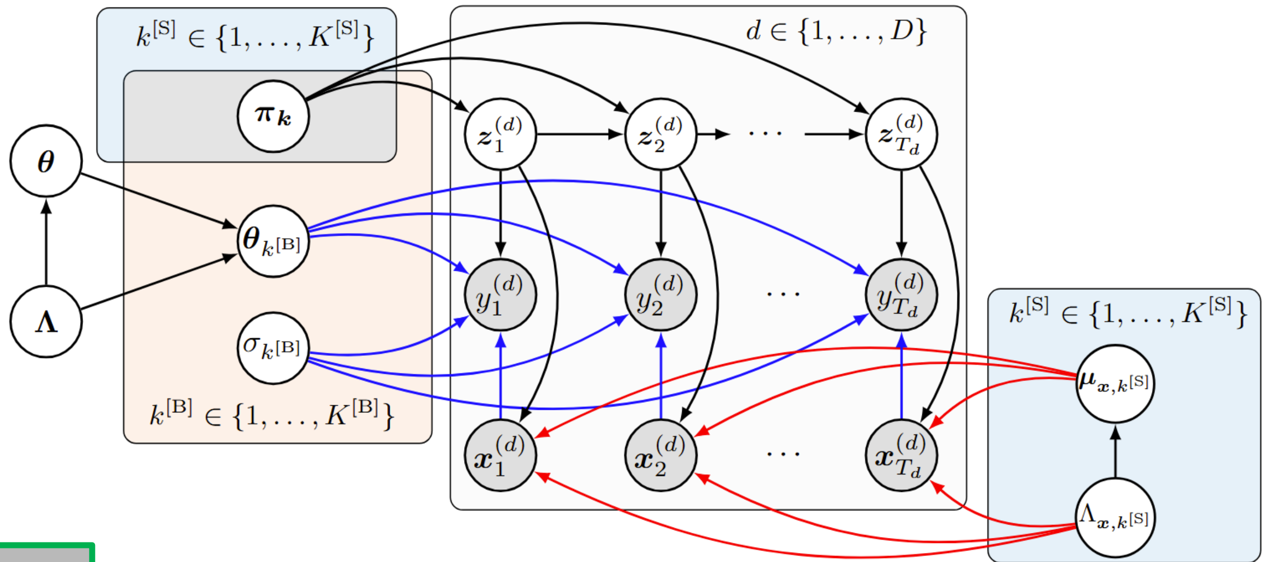


- Markovian regime switching
- Factorial latent states $\boldsymbol{z}_t \in \mathcal{Z} = \{1,\ldots,K^{[B]}\} \times \{1,\ldots,K^{[S]}\}$

[Chengyuan Zhang, Cathy Wu, and Lijun Sun. "Markov Regime-Switching Intelligent Driver Model for Interpretable Car-Following Behavior." *Preprint* (*under review*).]

Latent Variable Modeling – Hidden Markov Model

$$a(x,t) \approx f_{\mathrm{CF}}(x;\boldsymbol{\theta}_{z_t}), \quad z_t \sim \text{Markov Chain}$$



Posterior

$$\boldsymbol{z}^\star_{1:T}, \boldsymbol{\Omega}^\star = \arg\max_{\boldsymbol{z}_{1:T},\boldsymbol{\Omega}} \; p(\boldsymbol{z}_{1:T}, \boldsymbol{\Omega} \mid \boldsymbol{y}_{1:T}, \boldsymbol{x}_{1:T})$$

$$= \arg\max_{\boldsymbol{z}_{1:T},\boldsymbol{\Omega}} \; p(\boldsymbol{y}_{1:T}, \boldsymbol{x}_{1:T} \mid \boldsymbol{z}_{1:T}, \boldsymbol{\Omega}) \cdot p(\boldsymbol{z}_{1:T}) \cdot p(\boldsymbol{\Omega})$$

Likelihood **Priors**

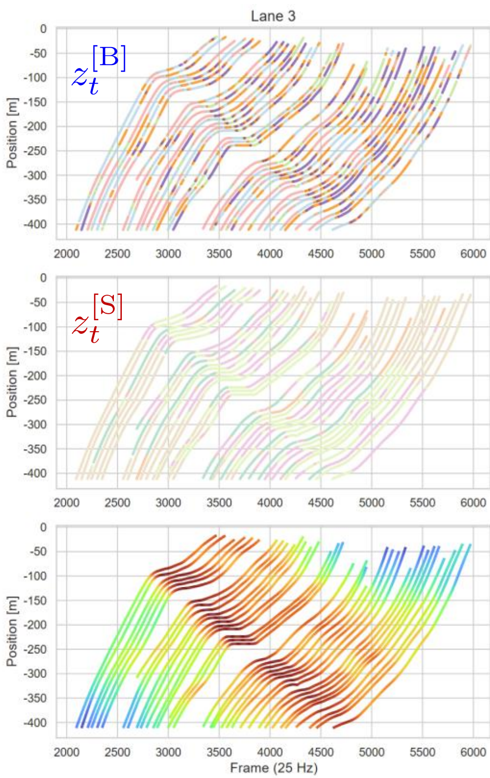
$$= \arg\max_{\boldsymbol{z}_{1:T},\boldsymbol{\Omega}} \; \prod_{t=1}^{T} p(y_t, \boldsymbol{x}_t \mid \boldsymbol{z}_t) \cdot p(\boldsymbol{z}_1) \prod_{t=2}^{T} p(\boldsymbol{z}_t \mid \boldsymbol{z}_{t-1}) \cdot p(\boldsymbol{\pi}) \cdot p(\sigma^2)$$

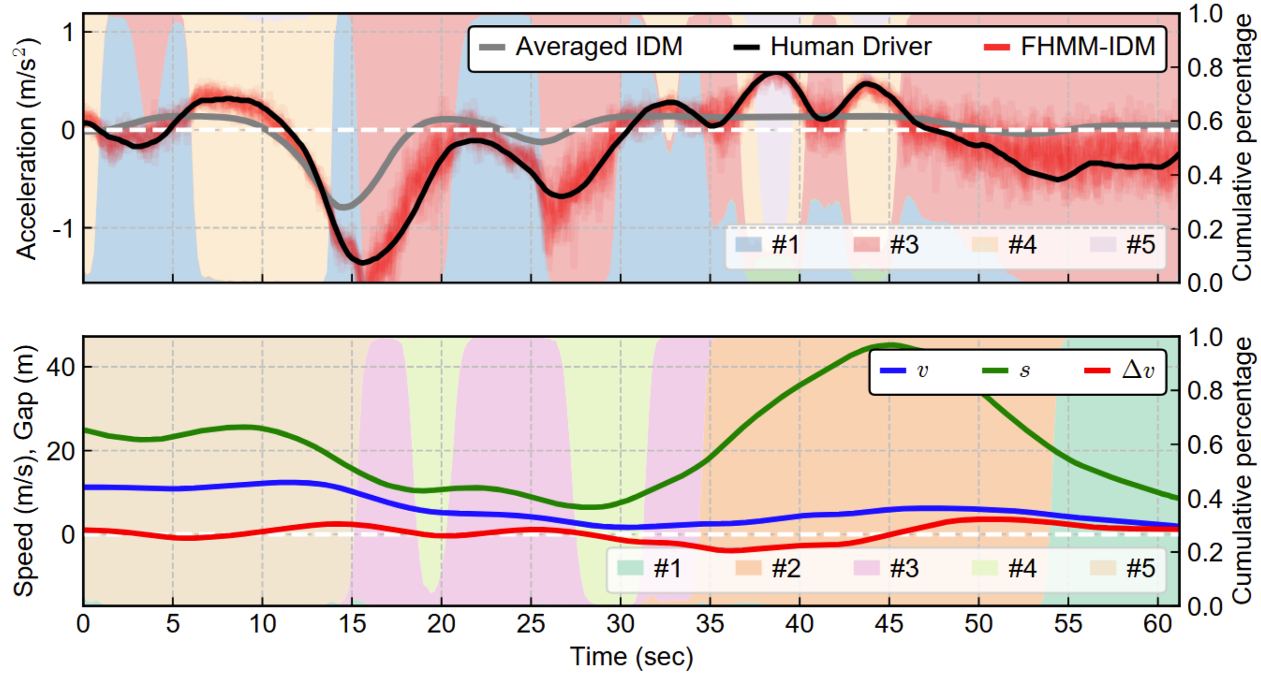
$$\cdot\, p(\boldsymbol{\Theta}) \cdot p(\boldsymbol{\mu}_{\boldsymbol{x}}, \boldsymbol{\Lambda}_{\boldsymbol{x}}) \cdot p(\boldsymbol{\mu}, \boldsymbol{\Lambda}).$$

Latent Variable Modeling – Hidden Markov Model

$$a(x,t) \approx f_{\mathrm{CF}}(x;\boldsymbol{\theta}_{z_t}), \quad z_t \sim \text{Markov Chain}$$

- FHMM-IDM: Identified Driving Regimes





Scenario #5 → #3 → #4 → #3 → #2 → #1.

Latent Variable Modeling – Hidden Markov Model

$$a(x,t) \approx f_{\mathrm{CF}}(x;\boldsymbol{\theta}_{z_t}), \quad z_t \sim \text{Markov Chain}$$

- **Solution B: build a better CFM by involving more information**

✓ This model: Memory; Heterogeneity; Stochasticity; Regime switching; Adaptation;

Feature/Model	**IDM**	**Bayesian IDM**	**GMM**	**HMM**	**HMM-GMM**	**HDP-HMM**	**NN (LSTM)**	**FHMM-IDM (Ours)**
Model Type	Deterministic	Probabilistic	Probabilistic	Probabilistic	Probabilistic	Probabilistic	Deep Learning	Probabilistic
Adaptivity[1]	✗	✗	✗	✓	✓	✓	✓	✓
Latent Behavior Type[2]	✗	✗	Discrete	Discrete	Discrete	Discrete	Continuous	Discrete
Latent Mode Cardinality[3]	-	-	Fixed	Fixed	Fixed	Infinite	Fixed	Factorial Fixed
Stochasticity	✗	✓	✓	✓	✓	✓	Implicit	✓
Parameter Estimation[4]	Heuristic	MCMC	EM/MCMC	EM/MCMC	EM/MCMC	EM/MCMC	Gradient descent	MCMC
Interpretability[5]	High	High	Moderate	Moderate	Moderate	Moderate	Low	High
Traffic Context Modeling[6]	✗	✗	✓(features)	✗	✓(features)	✓(implicit)	✓(learned)	✓(explicit)
Heterogeneity Handling[7]	Poor	Moderate	Moderate	Moderate	Moderate	Excellent	Excellent	Excellent
Data-driven Flexibility[8]	Low	Moderate	Moderate	Moderate	Moderate	High	High	High
Training Complexity[9]	Low	Moderate	Low	Moderate	Moderate/High	High	High	High

IDM: Treiber et al. (2000); Treiber and Helbing (2003); Treiber et al. (2006); Punzo et al. (2021); **Bayesian IDM**: Zhang and Sun (2024); Zhang et al. (2024b); **GMM**: Chen et al. (2023); Zhang et al. (2023, 2024a); **HMM**: Sathyanarayana et al. (2008); Aoude et al. (2012); Gadepally et al. (2013); Vaitkus et al. (2014); **HMM-GMM**: Wang et al. (2018b,a); **HDP-HMM**: Taniguchi et al. (2014); Zhang et al. (2021); Zou et al. (2022); **Neural Networks**: Wang et al. (2017); Zhu et al. (2018); Mo et al. (2021); Yao et al. (2025); Zhou et al. (2025);

[1] *Can the model dynamically adjust to changing behavior?*
[2] *Type of latent representation: discrete (mode switches) or continuous (trajectory embeddings).*
[3] *Whether the number of latent modes is fixed a priori or inferred.*
[4] *How model parameters are estimated: EM, gradient descent, MCMC, etc.*
[5] *Can latent states or parameters be interpreted as meaningful driving behavior?*
[6] *Whether traffic context (e.g., relative speed, gap) is explicitly used in latent modeling.*
[7] *Ability to capture driver-specific variation (e.g., hierarchical priors, class mixture).*
[8] *Model's ability to fit and learn from diverse and high-dimensional driving datasets.*
[9] *Overall training/inference complexity: data requirements, convergence cost, parallelism.*

Outline

✓ **Realistic simulation**

✓ **Improved interpretability**

with Solution A + Solution B

Nonstationary temporal correlations

- **Solution A assumes:**

$$a(x,t) = \boxed{f_{\mathrm{CFM}}(x;\boldsymbol{\theta})} + \boxed{\delta(t)} + \boxed{\epsilon_t,}\ \ \epsilon_t \overset{i.i.d.}{\sim} \mathcal{N}(0,\sigma_\epsilon^2)$$

- **MA-IDM assumes:**

$$a^{(t)} = \boxed{a_{\mathrm{IDM}}^{(t)}} + \boxed{a_{\mathrm{GP}}^{(t)}} + \boxed{\epsilon_t} \qquad \Rightarrow \boldsymbol{a}|\boldsymbol{i},\boldsymbol{\theta} \sim \mathcal{N}(\boxed{\boldsymbol{a}_{\mathrm{IDM}}}, \boxed{\boldsymbol{K}} + \boxed{\sigma_\epsilon^2 \boldsymbol{I}})$$

Homoscedasticity assumption with a stationary kernel. (inappropriate)

- **Nonstationary model assumes:**

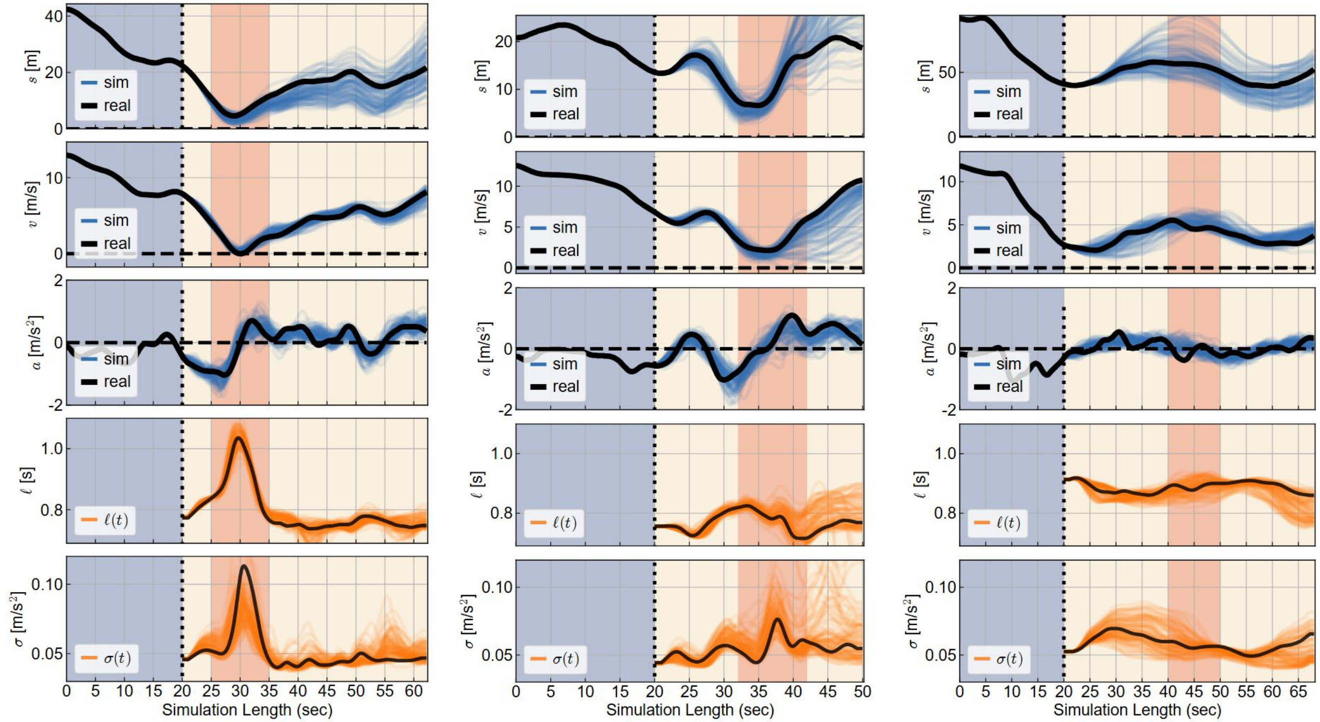
$$a^{(t)} = \boxed{a_{\mathrm{NN}}^{(t)}} + \boxed{a_{\mathrm{GP}}^{(t)}} + \boxed{\epsilon_t} \qquad \Rightarrow \boldsymbol{a}|\boldsymbol{i},\boldsymbol{\theta}_{\mathrm{NN}} \sim \mathcal{N}(\boxed{\boldsymbol{a}_{\mathrm{NN}}}, \boxed{\boldsymbol{K}} + \boxed{\sigma_\epsilon^2 \boldsymbol{I}})$$

Heteroscedasticity assumption with a nonstationary kernel (Gibbs kernel)

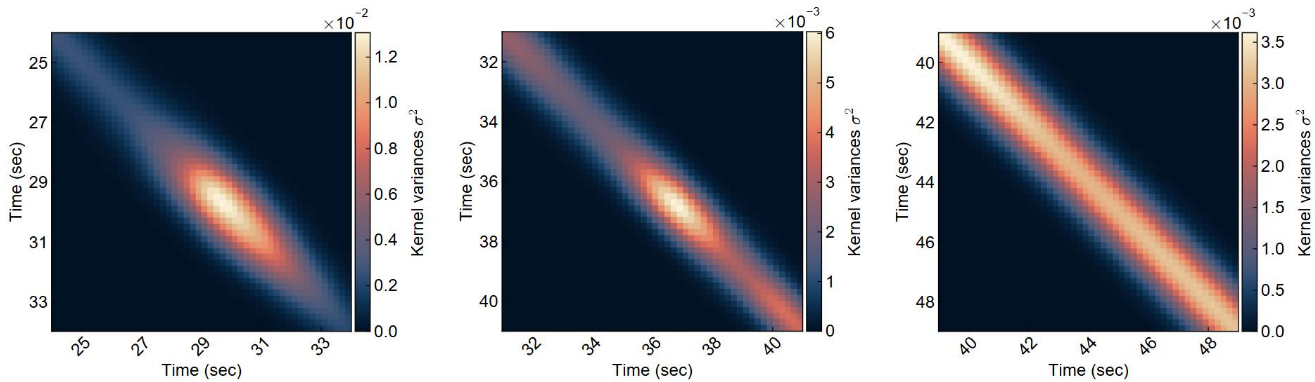
$$k_{\mathrm{Gibbs}}(t,t';\boldsymbol{\lambda}) := \sigma(t)\sigma(t')\sqrt{\frac{2\ell(t)\ell(t')}{\ell(t)^2+\ell(t')^2}}\exp\left(-\frac{(t-t')^2}{\ell(t)^2+\ell(t')^2}\right)$$

[Chengyuan Zhang, Zhengbing He, Cathy Wu, and Lijun Sun. (2025). Stochastic Modeling of Car-Following Behaviors with Nonstationary Temporal Correlations. *Preprint (under review)*.]

Nonstationary temporal correlations



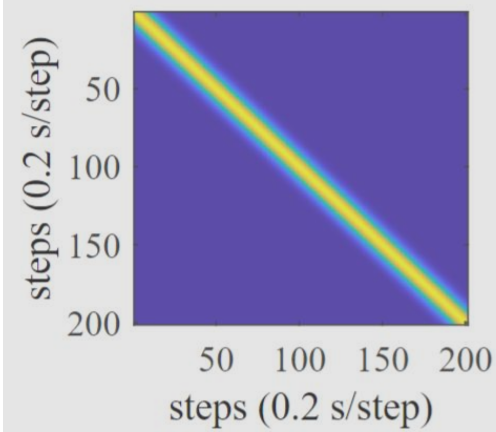
(a) Car-following Pair #1.　(b) Car-following Pair #2.　(c) Car-following Pair #3.



(d) Example $\boldsymbol{K}$ in Pair #1.　(e) Example $\boldsymbol{K}$ in Pair #2.　(f) Example $\boldsymbol{K}$ in Pair #3.

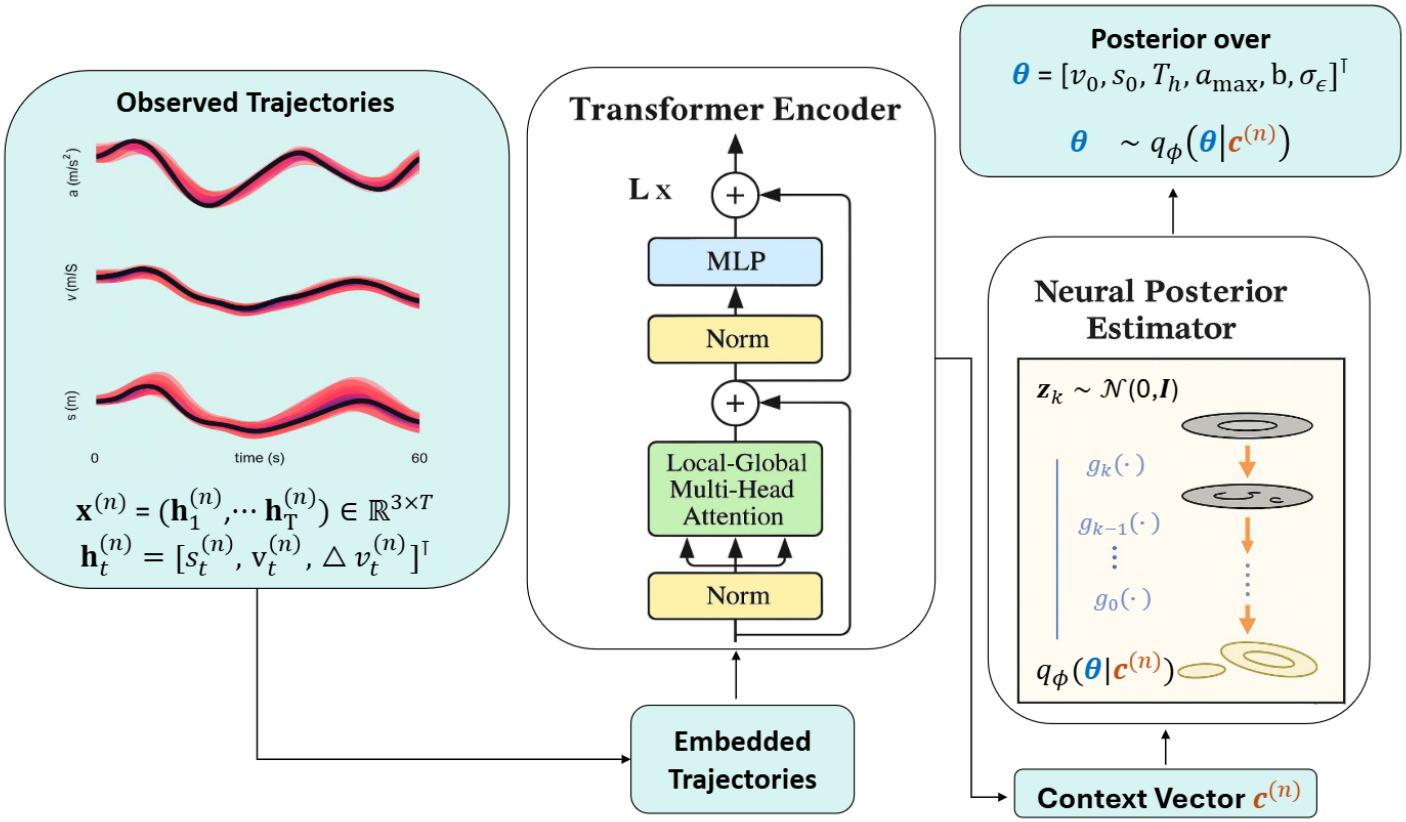
Lengthscale:
Smooth driving ↑
Abrupt transition ↓

Kernel variance:
Free/steady ↑
Safety-critical ↓



The RBF kernel matrix.

Stochastic Calibration via Simulation-Based Inference



Amortized SBI: input a trajectory, get an *instant* posterior over model parameters.

(likelihood-free calibration for black-box simulators, *e.g.*, SUMO)

[Menglin Kong, Chengyuan Zhang, Lijun Sun. "Stochastic Calibration of Car-Following Models via Simulation-Based Inference." To be presented on TRBAM 2026.] *(Unpublished Work)*

Outline

- **Challenge:** Traditional models learn a *one-to-one mapping*

$$f_{\mathrm{CF}} : (s_t, \Delta v_t, v_t) \mapsto a_t$$ (deterministic) ✗

but real drivers induce a *one-to-many mapping* with uncertainty

$$f_{\mathrm{CF}} : (s_t, \Delta v_t, v_t) \mapsto \{a_t^{(1)}, a_t^{(2)}, \dots\}$$ (stochastic) ✓

- **Solutions:**

① **Explicit Uncertainty Modeling** (continuous variability) — Solution A

$$a_t \approx f_{\mathrm{CF}}(\boldsymbol{x}_t; \boldsymbol{\theta}) + \delta_t,$$ (Zhang and Sun 2024, Zhang et al. 2024, Zhang et al. 2025a)

Table 1: Modeling of temporal correlations in my previous work.

Reference	$f_{\mathrm{CF}}(\boldsymbol{x}_t; \boldsymbol{\theta})$	δ_t
Zhang and Sun (2024)	IDM	Gaussian processes (GPs)
Zhang et al. (2024)	IDM	Autoregressive (AR) processes
Zhang et al. (2025a)	NN	nonstationary GPs

[Chengyuan Zhang and Lijun Sun. (2024). Bayesian calibration of the intelligent driver model. *IEEE Transactions on Intelligent Transportation Systems*.]
[Chengyuan Zhang, Wenshuo Wang, and Lijun Sun. Calibrating car-following models via Bayesian dynamic regression. (ISTTT25 Special Issue) Transportation Research Part C: Emerging Technologies 168 (2024): 104719.]
[Chengyuan Zhang, Zhengbing He, Cathy Wu, and Lijun Sun. (2025a). When Context Is Not Enough: Modeling Unexplained Variability in Car-Following Behavior. arXiv preprint arXiv:2507.07012 (*under review*).]

② **Latent Variable Modeling** (discrete variability) — Solution B

$$a_t \approx f_{\mathrm{CF}}(\boldsymbol{x}_t; \boldsymbol{\theta}_{z_t}), \; z_t \sim \begin{cases} \text{Gaussian Mixture} \\ \text{Markov Chain} \end{cases}$$

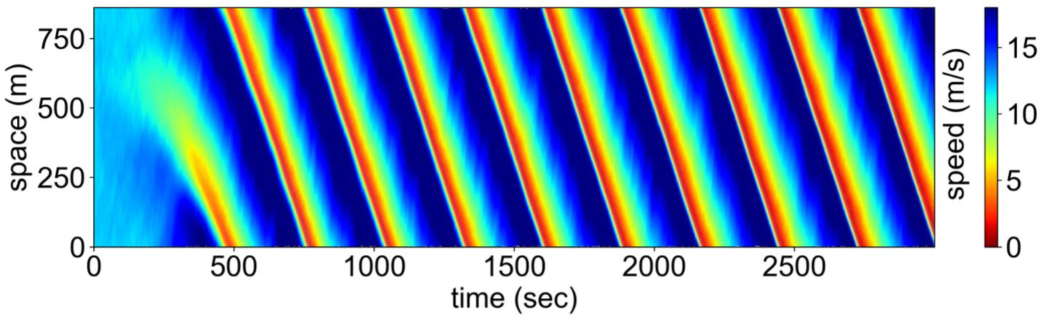
Gaussian Mixture: (*i.i.d.*) (Chen et al. 2023, Zhang et al. 2024)

Markov Chain: (*temporal dependence*) (Zhang et al. 2025b)

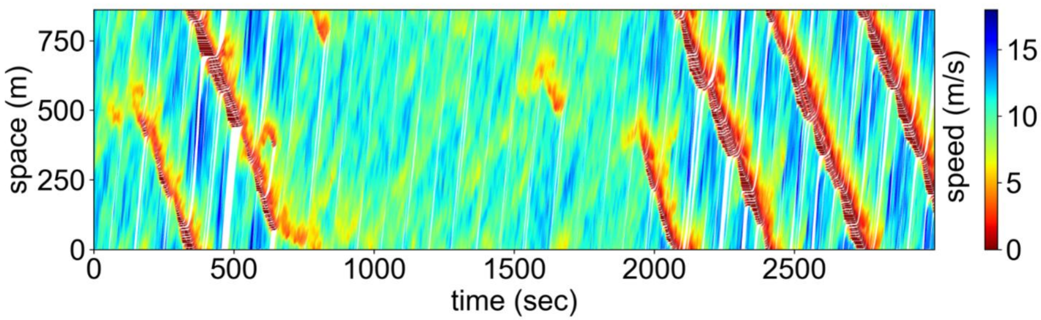
[Xiaoxu Chen, Chengyuan Zhang, Zhanhong Cheng, Yuang Hou, and Lijun Sun. A bayesian gaussian mixture model for probabilistic modeling of car-following behaviors. IEEE Transactions on Intelligent Transportation Systems 25, no. 6 (2023): 5880-5891.]
[Chengyuan Zhang, Kehua Chen, Meixin Zhu, Hai Yang, and Lijun Sun. Learning car-following behaviors using bayesian matrix normal mixture regression. In 2024 IEEE Intelligent Vehicles Symposium (IV), pp. 608-613. IEEE, 2024.]
[Chengyuan Zhang, Cathy Wu, and Lijun Sun. (2025b). Markov Regime-Switching Intelligent Driver Model for Interpretable Car-Following Behavior. arXiv preprint arXiv:2506.14762 (2025b). (*under review*) .]

→ stochastic, interpretable human-like simulators

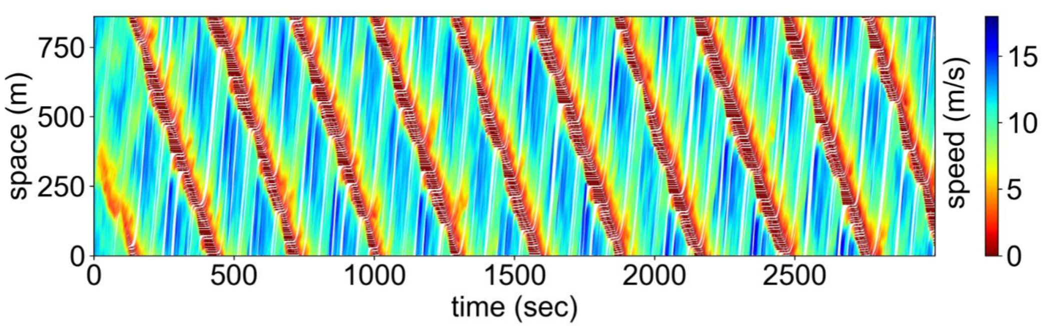
- **Outcome: stochastic, interpretable human-like simulators**



(a) Simulation with fixed IDM parameters and random white noise.



(b) Light traffic simulation with dynamic IDM ($p = 4$).

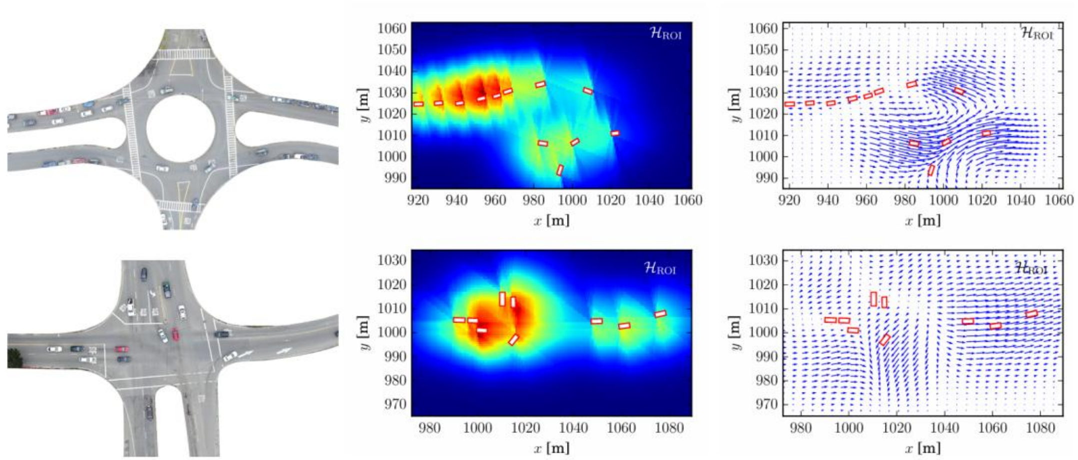


(c) Dense traffic simulation with dynamic IDM ($p = 4$).

Sugiyama experiment

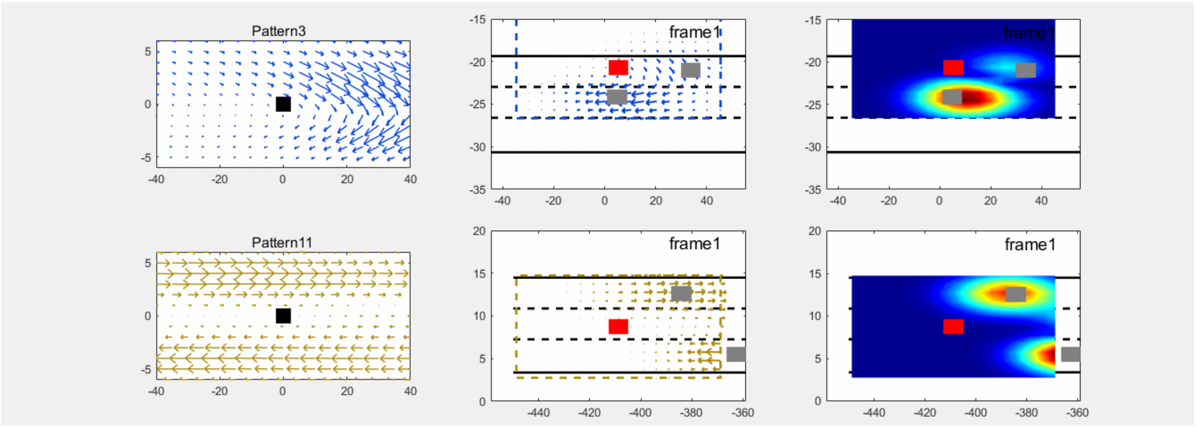
Other Interactive Scenarios (2019-2021)

(a) Roundabout & Intersection



[Wenshuo Wang, **Chengyuan Zhang**, Pin Wang, and Ching-Yao Chan. "Learning representations for multi-vehicle spatiotemporal interactions with semi-stochastic potential fields." In 2020 IEEE Intelligent Vehicles Symposium (IV), pp. 1935-1940. IEEE, 2020.]

(b) Lane Changing

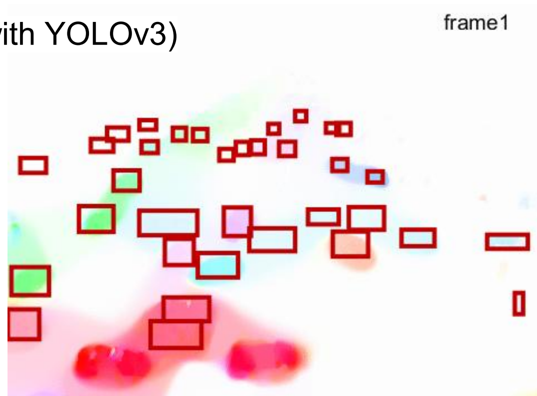


[**Chengyuan Zhang**, Jiacheng Zhu, Wenshuo Wang, and Junqiang Xi. "Spatiotemporal learning of multivehicle interaction patterns in lane-change scenarios." IEEE Transactions on Intelligent Transportation Systems 23, no. 7 (2021): 6446-6459.]

Other Interactive Scenarios (2019-2021)

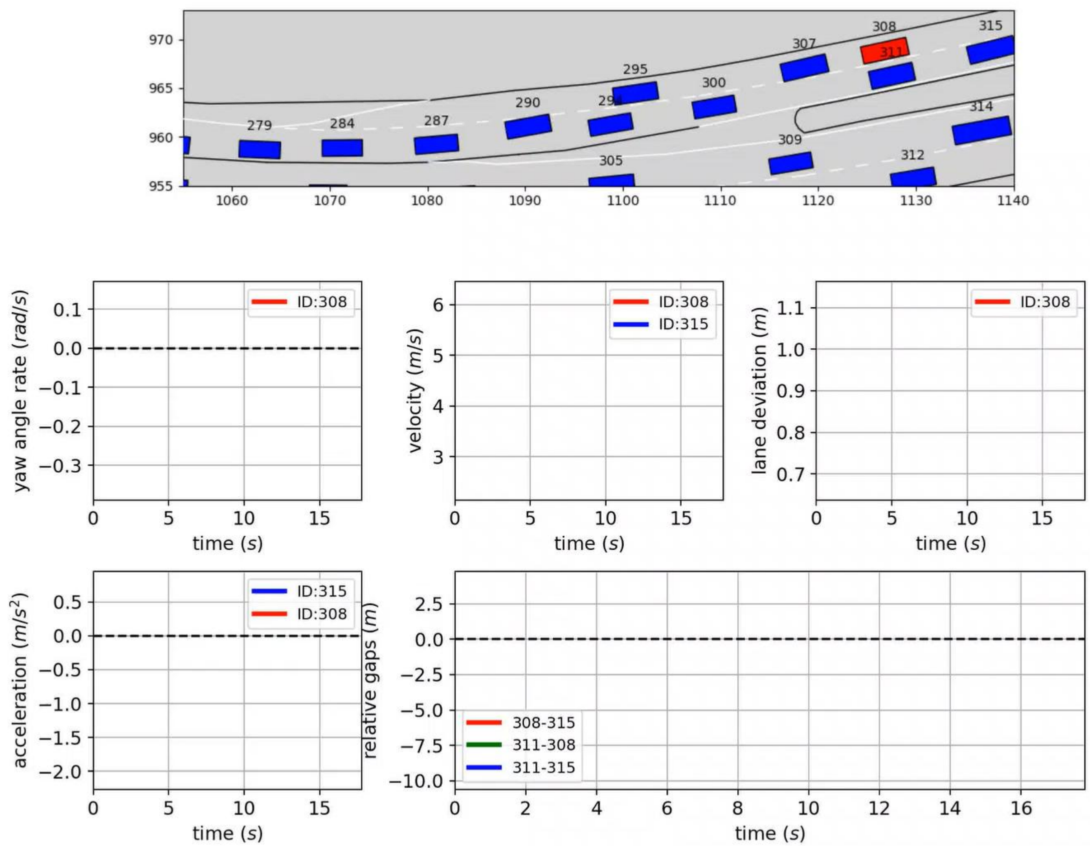
(c) Unsignalized Intersection

(with YOLOv3)



[**Chengyuan Zhang**, Jiacheng Zhu, Wenshuo Wang, and Ding Zhao. "A general framework of learning multi-vehicle interaction patterns from video." In 2019 IEEE Intelligent Transportation Systems Conference (ITSC), pp. 4323-4328. IEEE, 2019.]

(d) On-ramp merge in



[Unpublished Work.]

Discussion and takeaways

- Human Driving Behaviors Modeling and Stochastic Simulations
  - **Proper assumptions are important.**
    - Residuals are correlated --- we cannot use *i.i.d.* error assumption;
    - Heteroscedasticity assumption with a nonstationary kernel;
  - **Always think about “What is missing?”**
    - Positive / negative correlations;
    - Temporal structure;
  - **Social interaction is complex; even the simplest car-following behaviors remain under investigation.**

Related Publications

- **Paper:**

1. **Chengyuan Zhang**, Zhengbing He, Cathy Wu, and Lijun Sun. "When Context Is Not Enough: Modeling Unexplained Variability in Car-Following Behavior." *arXiv preprint arXiv:2507.07012* (2025).(NN with nonstationary GP)
2. **Chengyuan Zhang**, Cathy Wu, and Lijun Sun. "Markov Regime-Switching Intelligent Driver Model for Interpretable Car-Following Behavior." *arXiv preprint arXiv:2506.14762* (2025). (driving patterns with HMM)
3. **Chengyuan Zhang**, and Lijun Sun. "Bayesian calibration of the intelligent driver model." *IEEE Transactions on Intelligent Transportation Systems* 25, no. 8 (2024): 9308-9320. (IDM with GP)
4. **Chengyuan Zhang**, Wenshuo Wang, and Lijun Sun. "Calibrating car-following models via Bayesian dynamic regression." *(ISTTT25 Special Issue) Transportation Research Part C: Emerging Technologies* 168 (2024): 104719. (IDM with AR)
5. **Chengyuan Zhang**, Kehua Chen, Meixin Zhu, Hai Yang, and Lijun Sun. "Learning car-following behaviors using bayesian matrix normal mixture regression." In *2024 IEEE Intelligent Vehicles Symposium (IV)*, pp. 608-613. IEEE, 2024. (Mixture model with temporal structure)
6. **Chengyuan Zhang**, Rui Chen, Jiacheng Zhu, Wenshuo Wang, Changliu Liu, and Lijun Sun. "Interactive car-following: Matters but not always." In *2023 IEEE 26th International Conference on Intelligent Transportation Systems (ITSC)*, pp. 5120-5125. IEEE, 2023. (quantify interactions)
7. Xiaoxu Chen, **Chengyuan Zhang**, Zhanhong Cheng, Yuang Hou, and Lijun Sun. "A bayesian gaussian mixture model for probabilistic modeling of car-following behaviors." *IEEE Transactions on Intelligent Transportation Systems* 25, no. 6 (2023): 5880-5891. (driving patterns with GMM)
8. Wenshuo Wang, Letian Wang, **Chengyuan Zhang**, Changliu Liu, and Lijun Sun. "Social interactions for autonomous driving: A review and perspectives." Foundations and Trends® in Robotics 10, no. 3-4 (2022): 198-376. (review of social interactions)
9. **Chengyuan Zhang**, Jiacheng Zhu, Wenshuo Wang, and Junqiang Xi. "Spatiotemporal learning of multivehicle interaction patterns in lane-change scenarios." *IEEE Transactions on Intelligent Transportation Systems* 23, no. 7 (2021): 6446-6459. (driving patterns of lane-change)
10. Wenshuo Wang, **Chengyuan Zhang**, Pin Wang, and Ching-Yao Chan. "Learning representations for multi-vehicle spatiotemporal interactions with semi-stochastic potential fields." In *2020 IEEE Intelligent Vehicles Symposium (IV)*, pp. 1935-1940. IEEE, 2020. (intersection & roundabout)
11. **Chengyuan Zhang**, Jiacheng Zhu, Wenshuo Wang, and Ding Zhao. "A general framework of learning multi-vehicle interaction patterns from video." In *2019 IEEE Intelligent Transportation Systems Conference (ITSC)*, pp. 4323-4328. IEEE, 2019. (driving patterns of intersection)
12. Menglin Kong, **Chengyuan Zhang**, Lijun Sun. "Stochastic Calibration of Car-Following Models via Simulation-Based Inference." *To be presented on TRBAM 2026*. (SBI for calibration)

- **Code:**

1. https://github.com/Chengyuan-Zhang/IDM_Bayesian_Calibration
2. https://github.com/Chengyuan-Zhang/Gaussian_Velocity_Field

Thanks!
Questions?

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