25th INTERNATIONAL SYMPOSIUM on TRANSPORTATION and TRAFFIC THEORY

Calibrating Car-Following Models via Bayesian Dynamic Regression

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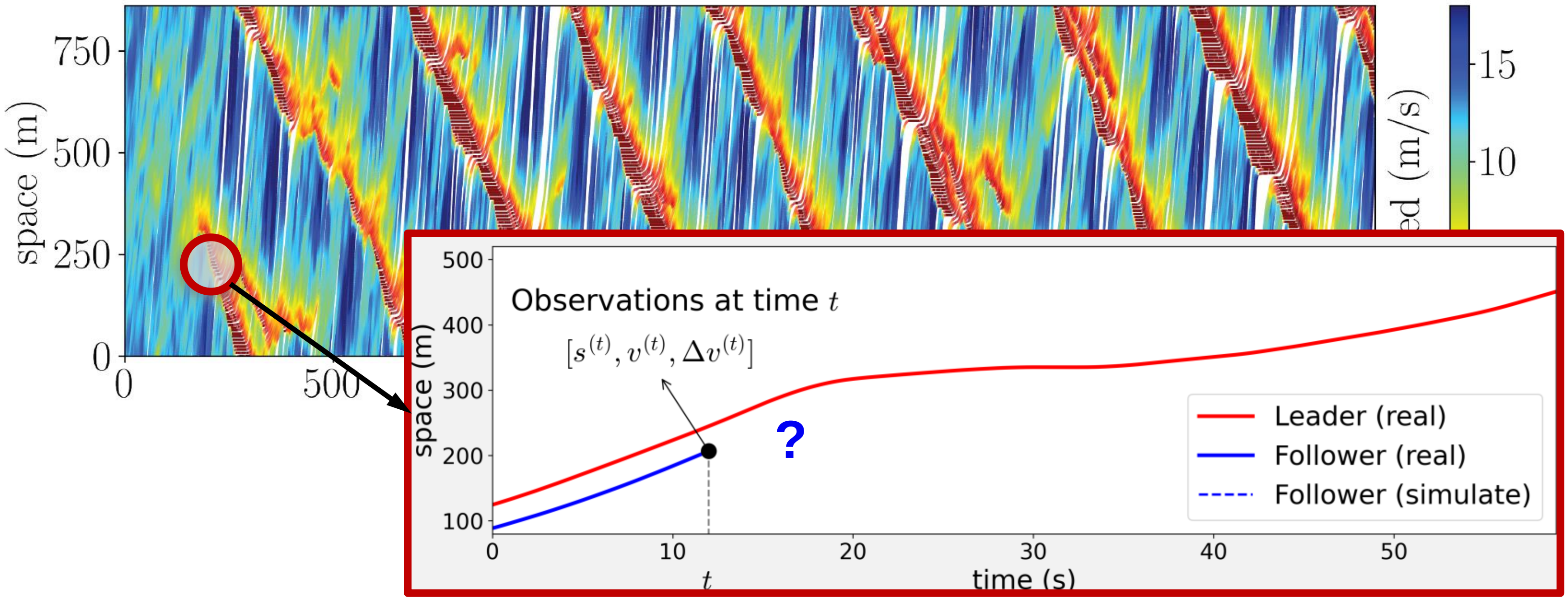
July 17, 2024

Motivation / background

- How would the vehicle react in response to the leading vehicle?



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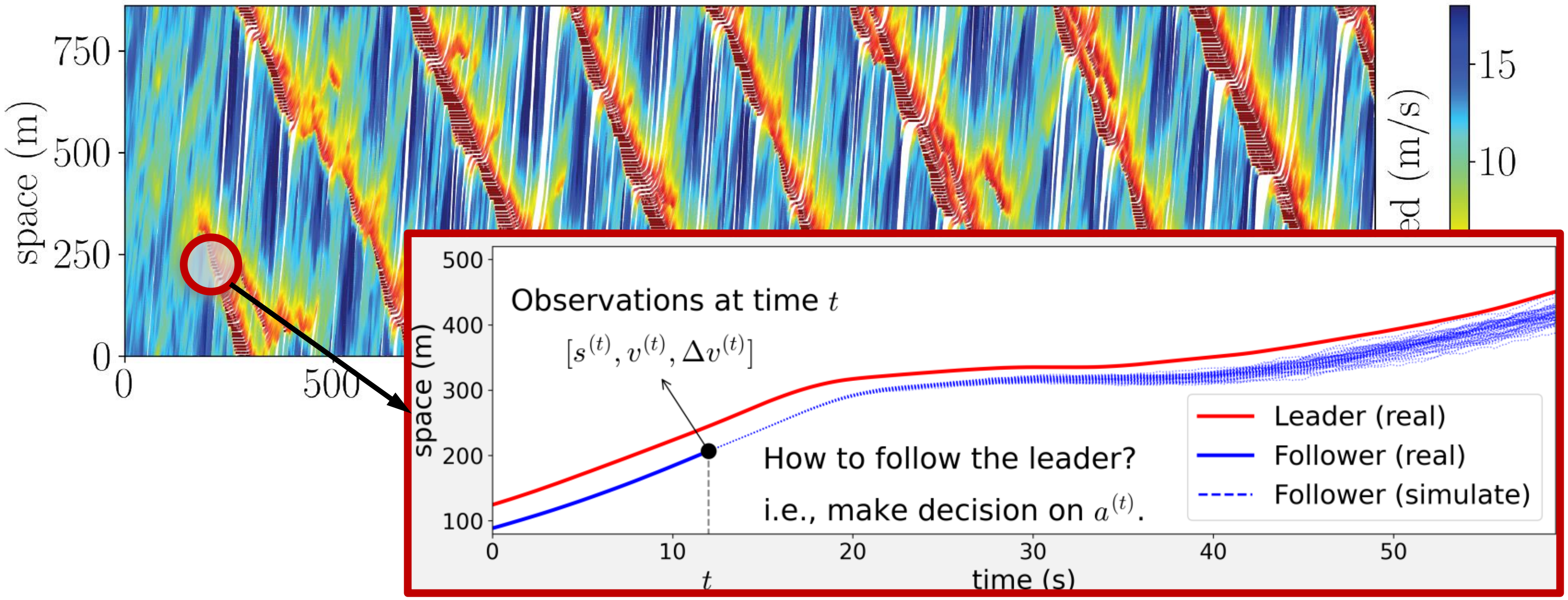


- What do we need from simulation?

Motivation / background

- The goal of traffic simulations:
  - **Past** : reproduce traffic phenomenon
  - **Future** : support the development and test of control algorithms
    - Connected and Automated Vehicle
    - Reinforcement learning for traffic control/management
    - Human drivers still involved
    - Safety, predictability, and uncertainty

- How do we introduce randomness?
  - x Deterministic car-following models (No)
  - ✓ **Human-driver car-following models** (Yes)

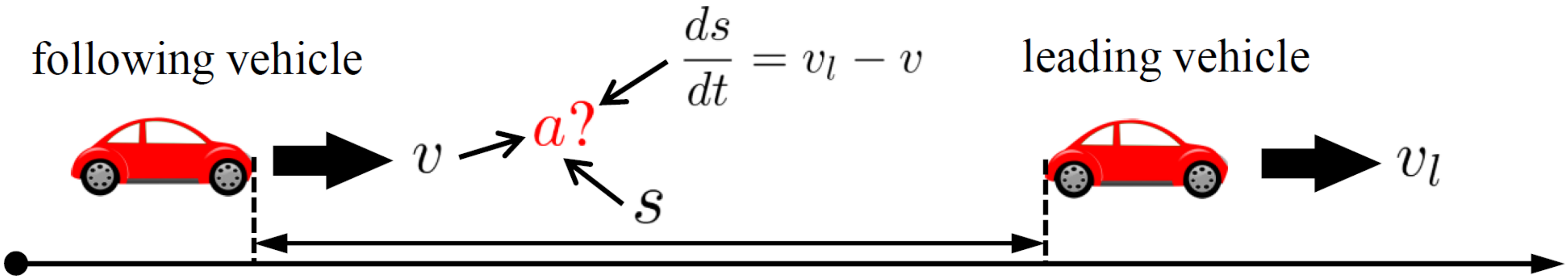
In this work, we are interested in:

- ➢ **How do we calibrate a human-driver car-following model?**
- ➢ **How do we simulate human-like car-following behaviors?**

Outline

- Intelligent driver model (IDM, as an example)
- Probabilistic modeling framework (Bayesian IDM, GP+IDM, AR+IDM)
- Numerical experiments for calibration
- Simulation
- Discussion

Intelligent driver model



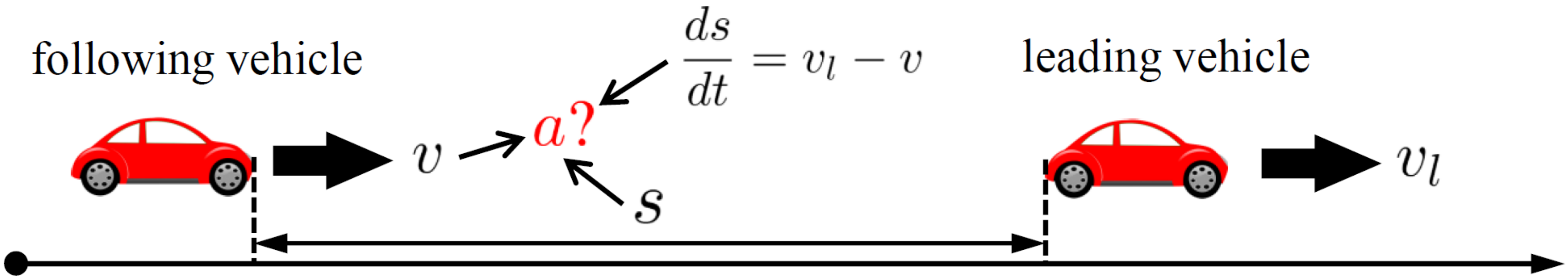
- **Intelligent Driver Model (IDM)** (Treiber et al. 2000)

$$a_{\mathrm{IDM}} = \alpha \left(1 - \left(\frac{v}{v_0}\right)^{\delta} - \left(\frac{s^*(v, \Delta v)}{s}\right)^2\right)$$

$$s^*(v, \Delta v) = s_0 + s_1 \sqrt{\frac{v}{v_0}} + v\,T + \frac{v\,\Delta v}{2\sqrt{\alpha\,\beta}}$$

where $v_0, s_0, T, \alpha, \beta$ and δ are model parameters,we denote these parameters as a vector $\boldsymbol{\theta} = [v_0, s_0, T, \alpha, \beta] \in \mathbb{R}^5$, and we fix $\delta = 4$.

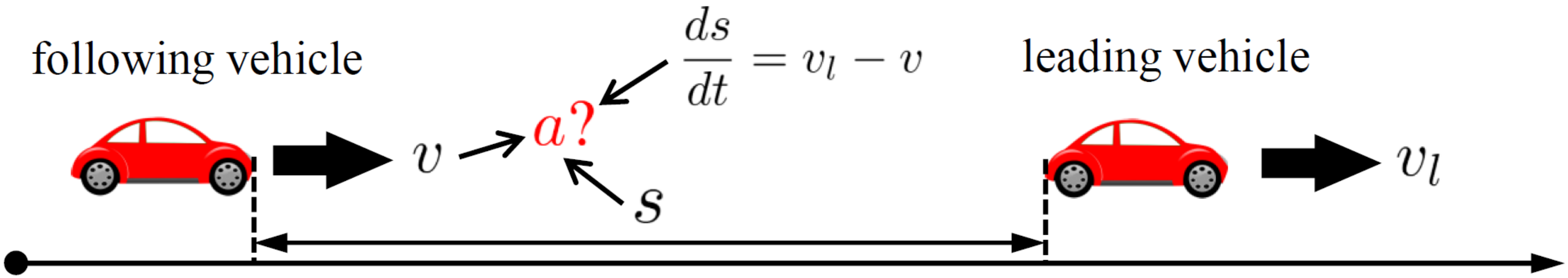
Intelligent driver model



- IDM assumes: $a^{(t)} = a_{\text{IDM}}^{(t)} + \epsilon_t, \ \epsilon_t \overset{i.i.d.}{\sim} \mathcal{N}(0, \sigma_\epsilon^2).$
- Calibration by MLE: $\max \text{ likelihood} = \prod_{t=1}^{T} \mathcal{N}(\hat{a}^{(t)} | a_{\text{IDM}}^{(t)}, \sigma_\epsilon^2)$
- Loss function in literature (Punzo et al. 2000):

$$\min \mathcal{L}, \ \mathcal{L} = \frac{1}{T}\sum_{t=1}^{T}(a^{(t)} - \hat{a}^{(t)})^2 + \frac{\alpha}{T}\sum_{t=1}^{T}(v^{(t)} - \hat{v}^{(t)})^2 + \frac{\beta}{T}\sum_{t=1}^{T}(x^{(t)} - \hat{x}^{(t)})^2$$

Intelligent driver model



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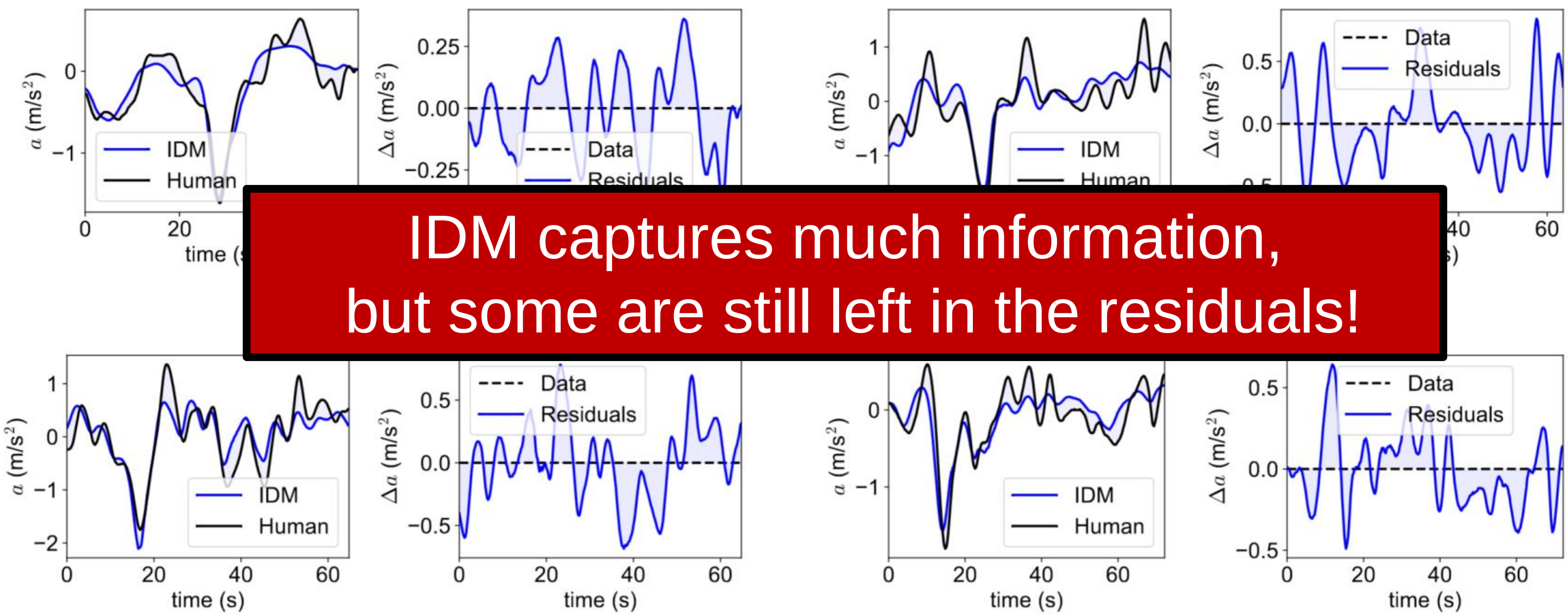
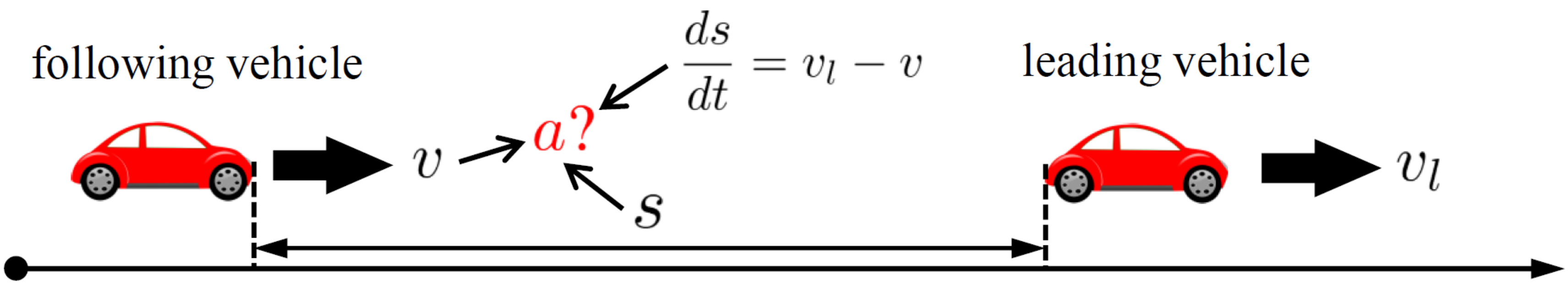


IDM captures much information,
but some are still left in the residuals!

Intelligent driver model

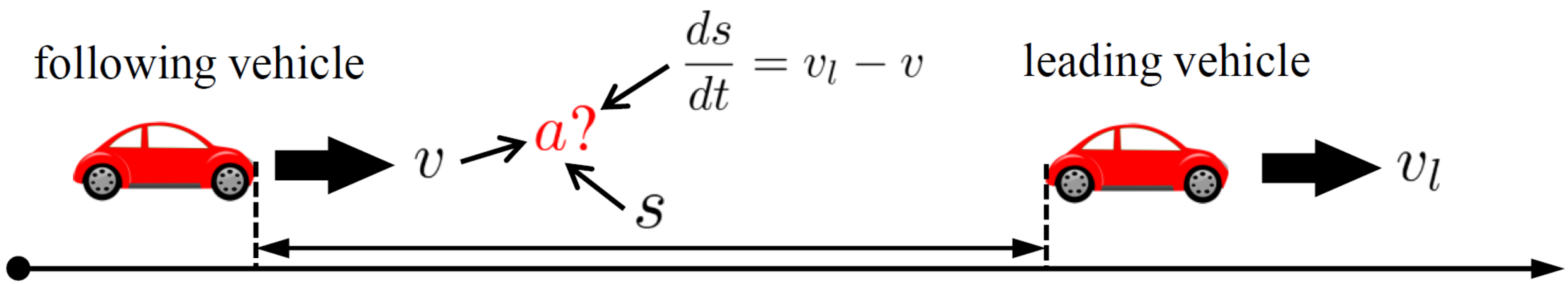


- For a human-driver CF model, what do we miss?
  - Reaction time / action inertia (of the *drivers*)
  - Brake light signals (from the *leaders*)
  - Nudging behaviors (from the *followers*)
  - Temporally correlated errors / Time delay (from the *model* aspect)
  - Heterogeneity of drivers (from the *model* aspect)

- IDM as a **parsimonious** model can hardly explain all the variation in the data; as a result, the residual terms are **serially correlated**;

- **How do we integrate these factors when calibrating IDM?**

Intelligent driver model



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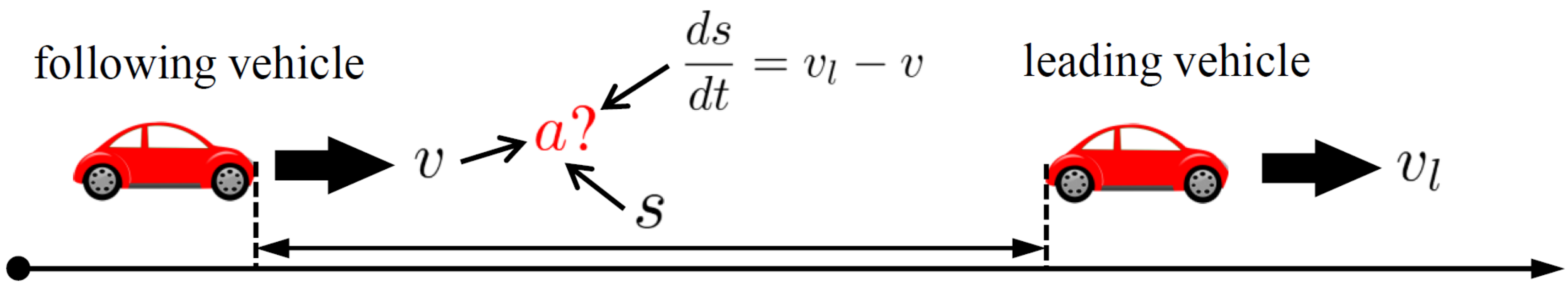
Intelligent driver model



- **Real process**: $a(x,t) = a(x;\boldsymbol{\theta}) + \delta(t) + \epsilon, \; \epsilon \overset{i.i.d.}{\sim} \mathcal{N}(0, \sigma_\epsilon^2)$
  (Kennedy and O'Hagan. 2001)

- **IDM**: $a(x,t) = a_{\mathrm{IDM}}(x;\boldsymbol{\theta}) + \epsilon, \; \epsilon \overset{i.i.d.}{\sim} \mathcal{N}(0, \sigma_\epsilon^2)$
  (Treiber et al. 2000)

<u>Missed the temporal part</u> $\delta(t)$

TO-DO:

- Consider $\delta(t)$ in calibration;
- Model $\delta(t+1)|\delta(t)$ in simulations.

Memory-Augmented IDM (MA-IDM) (Zhang and Sun 2024)

How to model $\delta(t)$ and $\delta(t+1)|\delta(t)$?

- **Real process:**

$$a(x,t) = a(x;\boldsymbol{\theta}) + \delta(t) + \epsilon,\quad \epsilon \overset{i.i.d.}{\sim} \mathcal{N}(0,\sigma_\epsilon^2)$$

- **IDM assumes:**

$$a^{(t)} = a_{\mathrm{IDM}}^{(t)} + \epsilon_t,\quad \epsilon_t \overset{i.i.d.}{\sim} \mathcal{N}(0,\sigma_\epsilon^2) \quad \Rightarrow \boldsymbol{a}|\boldsymbol{i},\boldsymbol{\theta} \sim \mathcal{N}(\boldsymbol{a}_{\mathrm{IDM}}, \sigma_\epsilon^2 \boldsymbol{I})$$

Vector form

- **MA-IDM assumes:**

$$a^{(t)} = a_{\mathrm{IDM}}^{(t)} + a_{\mathrm{GP}}^{(t)} \quad \Rightarrow \boldsymbol{a}|\boldsymbol{i},\boldsymbol{\theta} \sim \mathcal{N}(\boldsymbol{a}_{\mathrm{IDM}}, \boldsymbol{K} + \sigma_\epsilon^2 \boldsymbol{I})$$

residuals

where ***K*** is a kernel matrix .

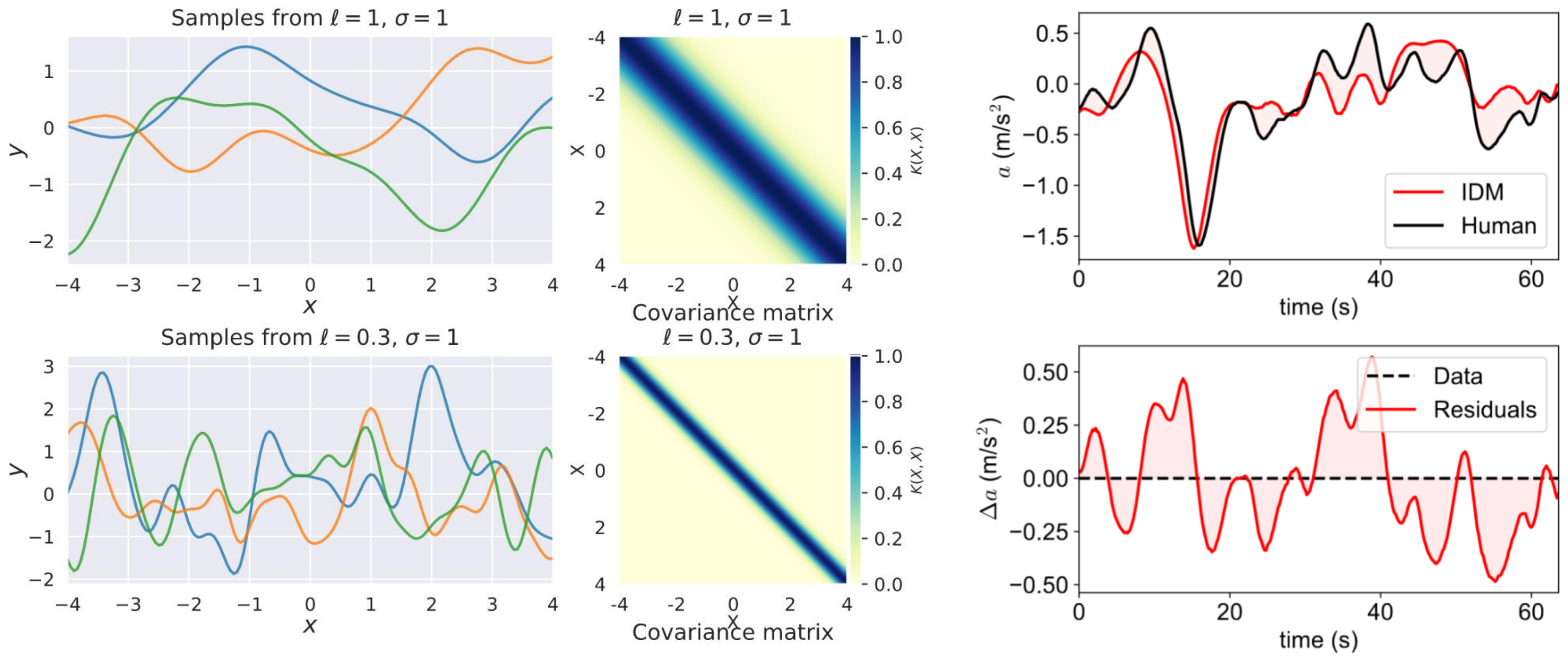
Memory-Augmented IDM (MA-IDM) (Zhang and Sun 2024)

- **MA-IDM assumes:**

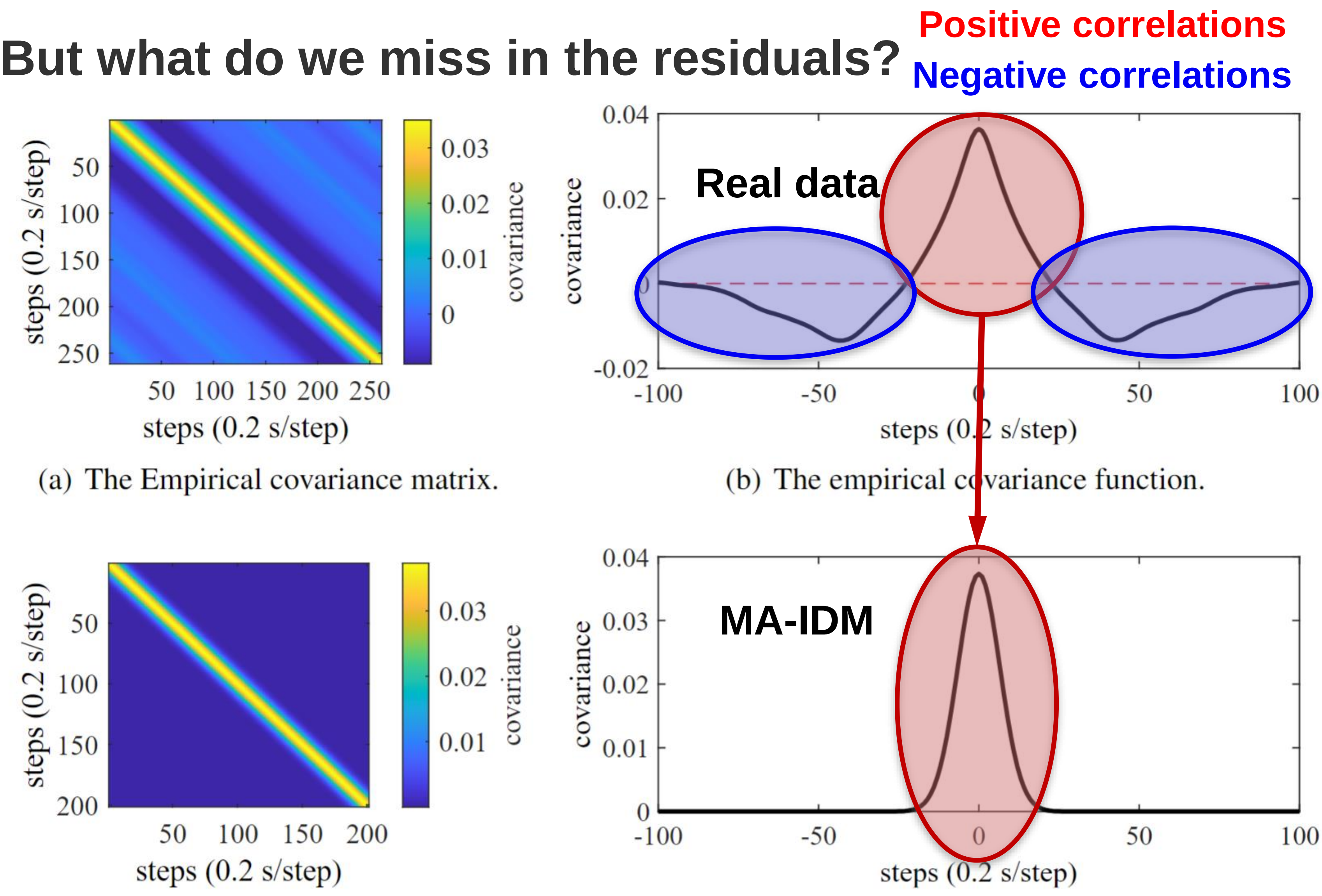
$$a^{(t)} = a_{\mathrm{IDM}}^{(t)} + \boxed{a_{\mathrm{GP}}^{(t)}} \quad \Rightarrow \boldsymbol{a} | \boldsymbol{i}, \boldsymbol{\theta} \sim \mathcal{N}(\boldsymbol{a}_{\mathrm{IDM}}, \boxed{\boldsymbol{K}})$$

residuals where ***K*** is a kernel matrix .

- Gaussian processes



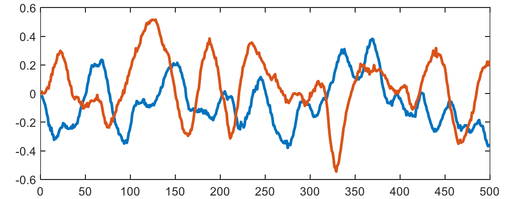
But what do we miss in the residuals?



(a) The Empirical covariance matrix.
(b) The empirical covariance function.
(c) The RBF kernel matrix.
(d) The RBF kernel function.

Dynamic IDM (AR+IDM)

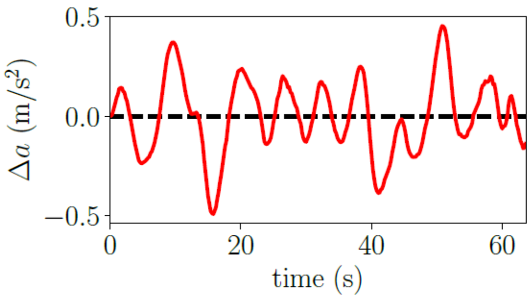
How to model $\delta(t)$ and $\delta(t+1)|\delta(t)$?



Two random series generated by AR(4)

- **Real process:**

$$a(x,t) = a(x;\boldsymbol{\theta}) + \delta(t) + \epsilon, \quad \epsilon \overset{i.i.d.}{\sim} \mathcal{N}(0, \sigma_\epsilon^2)$$



- **Dynamic IDM assumes:**

Autoregressive (AR) processes

$$a_d^{(t)} = \mathrm{IDM}_d^{(t)} + \varepsilon_d^{(t)},$$
$$\varepsilon_d^{(t)} = \rho_{d,1}\varepsilon_d^{(t-1)} + \rho_{d,2}\varepsilon_d^{(t-2)} + \cdots + \rho_{d,p}\varepsilon_d^{(t-p)} + \eta_d^{(t)},$$
$$\eta^{(t)} \overset{i.i.d.}{\sim} \mathcal{N}(0, \sigma_\eta^2).$$

ADVANTAGE: It involves rich information from **several historical steps** instead of using only one step.

Calibration

Bayesian: p(params|data) ∝ p(data |params) p(params)

- **Bayesian IDM**

$$\boldsymbol{\sigma}_0 \overset{\text{i.i.d.}}{\sim} \text{Exp}(\lambda_0),$$
$$\boldsymbol{\Sigma} \sim \text{LKJCholeskyCov}(\eta, \boldsymbol{\sigma}_0),$$
$$\ln(\boldsymbol{\theta}) \sim \mathcal{N}(\ln(\boldsymbol{\theta}_{\text{rec}}), \boldsymbol{\Sigma}_0),$$
$$\sigma_\eta \sim \text{Exp}(\lambda_\eta),$$
$$\textbf{for driver } d = 1, \ldots, D:$$
$$\ln(\boldsymbol{\theta}_d) \sim \mathcal{N}(\ln(\boldsymbol{\theta}), \boldsymbol{\Sigma}),$$
$$\textbf{for time } t = t_0, \ldots, t_0 + (T_d - 1)\Delta t:$$
$$\hat{a}_d^{(t)} | \boldsymbol{h}_d^{(t)}, \boldsymbol{\theta}_d \overset{\text{i.i.d.}}{\sim} \mathcal{N}(\text{IDM}_d^{(t)}, \sigma_\eta^2)$$

- **MA-IDM (GP+IDM)**

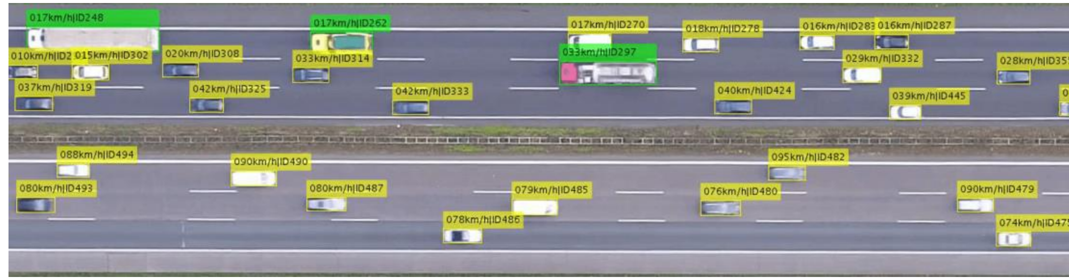
$$\sigma_k \sim \text{Exp}(\lambda_k),$$
$$\ln(\ell) \sim \mathcal{N}(\ln(\mu_\ell), \sigma_{\ell_0}^2),$$
$$\textbf{for driver } d = 1, \ldots, D:$$
$$\ln(\sigma_{k,d}) \sim \mathcal{N}(\ln(\sigma_k), \sigma_\sigma^2) \in \mathbb{R},$$
$$\ln(\ell_d) \sim \mathcal{N}(\ln(\ell), \sigma_\ell^2) \in \mathbb{R},$$
$$\boldsymbol{a}_d | \boldsymbol{h}_d, \boldsymbol{\theta}_d \overset{\text{i.i.d.}}{\sim} \mathcal{N}(\textbf{IDM}_d, \boldsymbol{K}_d).$$

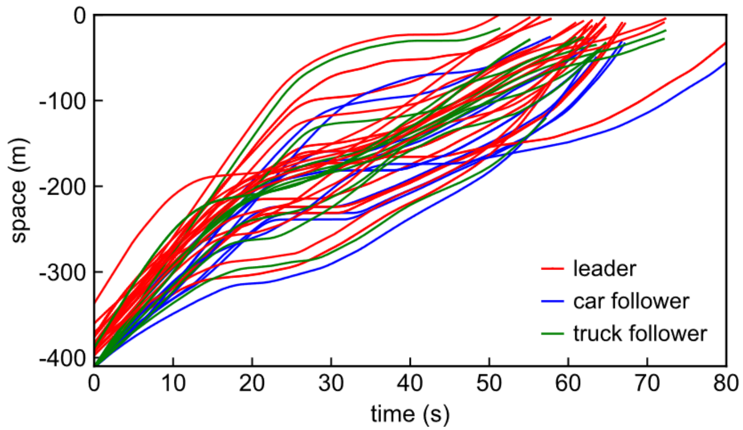
- **Dynamic IDM (AR+IDM)**

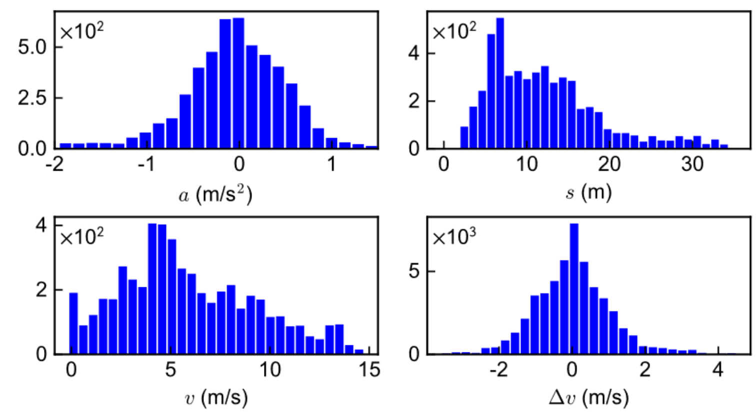
$$\sigma_\eta \sim \text{Exp}(\lambda_\eta),$$
$$\boldsymbol{\rho} \sim \mathcal{N}(\boldsymbol{0}, \sigma_{\rho_0}^2 \boldsymbol{I}),$$
$$\textbf{for driver } d = 1, \ldots, D:$$
$$\boldsymbol{\rho}_d \sim \mathcal{N}(\boldsymbol{\rho}, \sigma_\rho^2 \boldsymbol{I}),$$
$$\textbf{for time } t = t_0, \ldots, t_0 + (T_d - 1)\Delta t:$$
$$a_d^{(t)} | \boldsymbol{h}_d^{(t)}, \boldsymbol{\theta}_d \overset{\text{i.i.d.}}{\sim} \mathcal{N}\left(\text{IDM}_d^{(t)} + \sum_{k=1}^{p} \rho_{d,k} \left(a_d^{(t-k)} - \text{IDM}_d^{(t-k)}\right), \sigma_\eta^2\right),$$

Experiments – Car-Following Data Extraction

- **HighD** dataset; (Krajewski et al. 2018)
- **20** leader-follower pairs.







- Intelligent Driver Model

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$$s^*(v,\Delta v) = s_0 + s_1\sqrt{\frac{v}{v_0}} + vT + \frac{v\,\Delta v}{2\sqrt{\alpha\,\beta}}$$

Must provide enough info to calibrate IDM!

- v_0: free-flow;
- s_0 and T: steady following;
- α: freely accelerating data;
- β: approaching (with braking).

Experiments – Identified Parameters

The length scale ≈ 1.5 sec → capture positive correlations within 4~5 sec (3-sigma in Normal distribution).

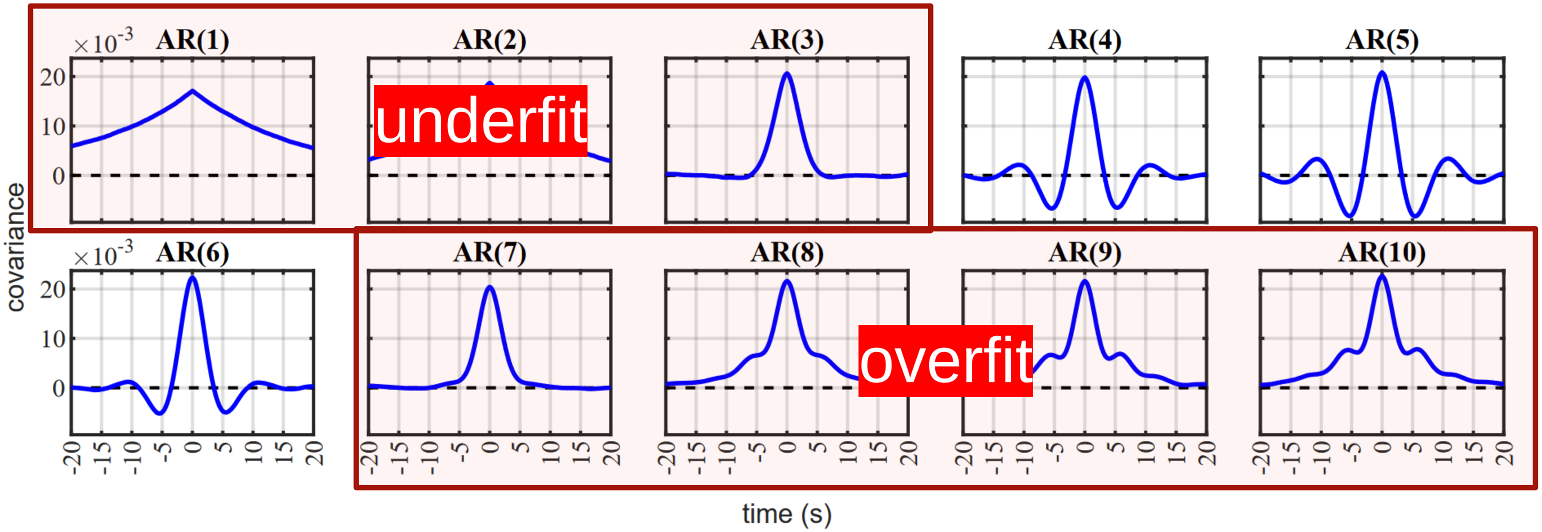
Table 1: Posterior Mean of Model Parameters.

Models	$\boldsymbol{\theta} = [v_0, s_0, T, \alpha, \beta]$	σ_η	ρ
MA-IDM	$[16.919, 3.538, 1.183, 0.553, 2.147]$	/	$(\sigma_k = 0.202, \ell = 1.44$ s)
Bayesian IDM ($p = 0$)	$[21.090, 3.724, 0.946, 0.518, 1.542]$	0.240	/
Dynamic IDM ($p = 1$)	$[29.738, 3.220, 1.186, 0.769, 4.130]$	0.019	$[0.989]$
Dynamic IDM ($p = 2$)	$[27.592, 3.367, 1.191, 0.741, 3.483]$	0.019	$[1.234, -0.247]$
Dynamic IDM ($p = 3$)	$[25.004, 2.974, 1.206, 0.811, 2.442]$	0.017	$[1.123, 0.425, -0.572]$
Dynamic IDM ($p = 4$)	$[26.181, 2.850, 1.222, 0.811, 3.145]$	0.016	$[0.901, 0.590, -0.149, -0.377]$
Dynamic IDM ($p = 5$)	$[27.099, 2.843, 1.235, 0.813, 3.422]$	0.016	$[0.874, 0.580, -0.105, -0.315, -0.071]$
Dynamic IDM ($p = 6$)	$[28.089, 2.702, 1.259, 0.826, 3.325]$	0.015	$[0.902, 0.632, -0.100, -0.427, -0.217, 0.181]$
Dynamic IDM ($p = 7$)	$[28.574, 2.594, 1.276, 0.817, 3.439]$	0.014	$[0.866, 0.690, -0.001, -0.413, -0.378, -0.032, 0.248]$
Dynamic IDM ($p = 8$)	$[28.446, 2.573, 1.264, 0.796, 3.805]$	0.014	$[0.816, 0.700, 0.075, -0.331, -0.381, -0.172, 0.080, 0.200]$
Dynamic IDM ($p = 9$)	$[29.675, 2.641, 1.265, 0.776, 4.452]$	0.014	$[0.794, 0.694, 0.093, -0.295, -0.351, -0.181, 0.016, 0.126, 0.090]$
Dynamic IDM ($p = 10$)	$[28.769, 2.739, 1.243, 0.763, 4.916]$	0.014	$[0.795, 0.694, 0.090, -0.295, -0.346, -0.178, 0.014, 0.121, 0.085, 0.007]$

* Recommendation values (Treiber et al., 2000): $\boldsymbol{\theta}_{\text{rec}} = [33.3, 2.0, 1.6, 1.5, 1.67]$.

MA-IDM

Experiments – Identified AR Parameters



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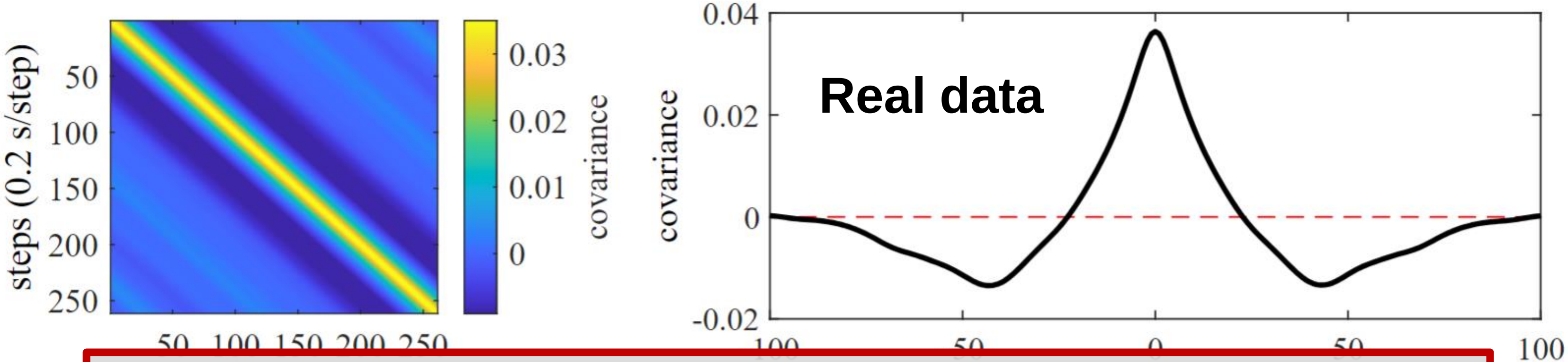
Dynamic IDM ($p=10$)	$[28.769, 2.739, 1.243, 0.763, 4.916]$	0.014	$[0.795, 0.694, 0.090, -0.295, -0.346, -0.178, 0.014, 0.121, 0.085, 0.007]$

* Recommendation values (Treiber et al., 2000): $\boldsymbol{\theta}_{\text{rec}} = [33.3, 2.0, 1.6, 1.5, 1.67]$.

Experiments – Identified AR Parameters



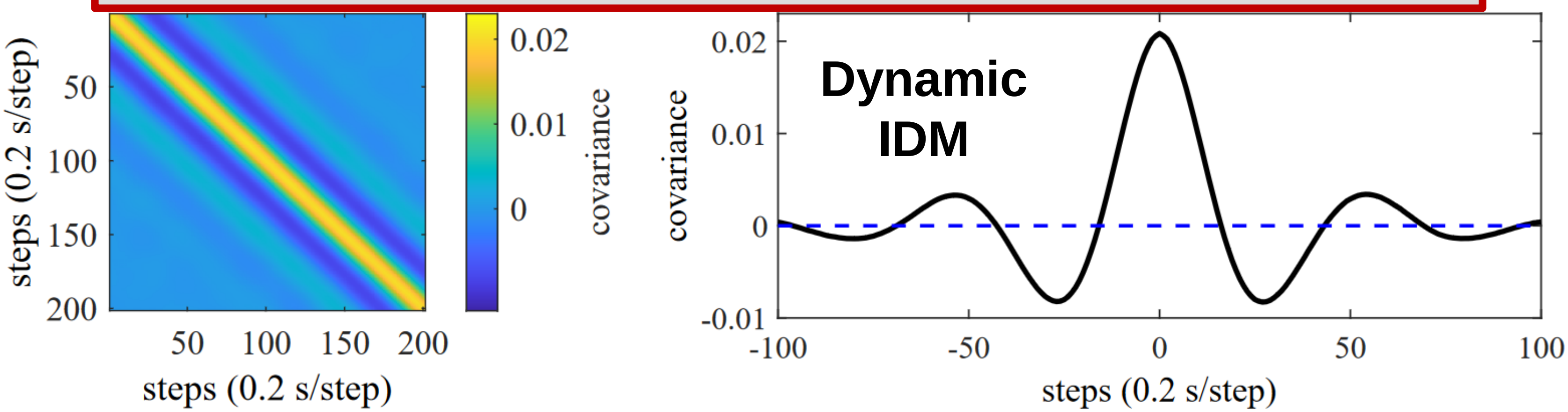
steps (0.2 s/step)

(a) The Empirical covariance matrix.

steps (0.2 s/step)

(b) The empirical covariance function.

Positive correlations: up to 5 s
Negative correlations: 5~10 s

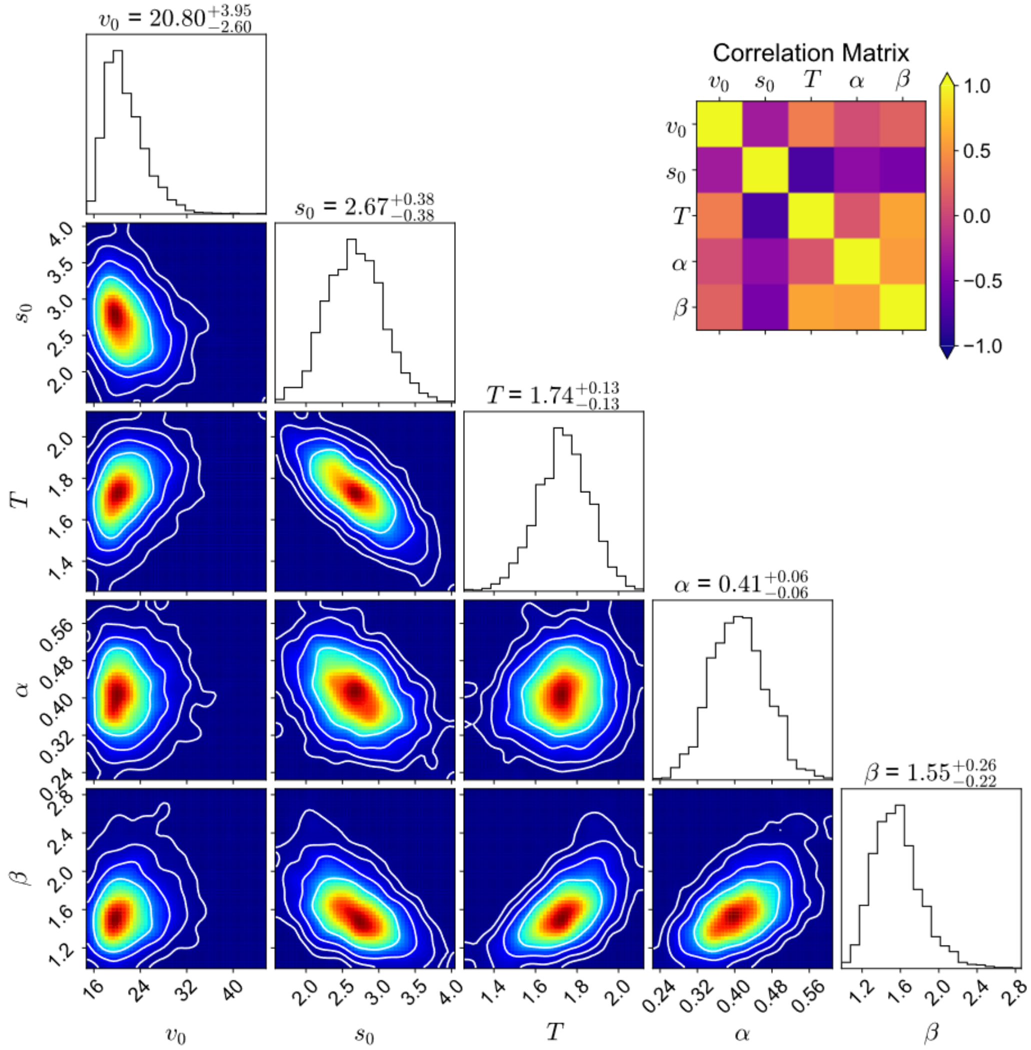


(e) The AR(5) covariance matrix.

(f) The AR(5) covariance functions.

$\boldsymbol{\rho} = [0.874, 0.580, -0.105, -0.315, -0.071]$

Experiments – Identified IDM Parameters



We can draw samples (IDM parameters) from the posterior distributions!!

Simulations – Deterministic *v.s.* Stochastic

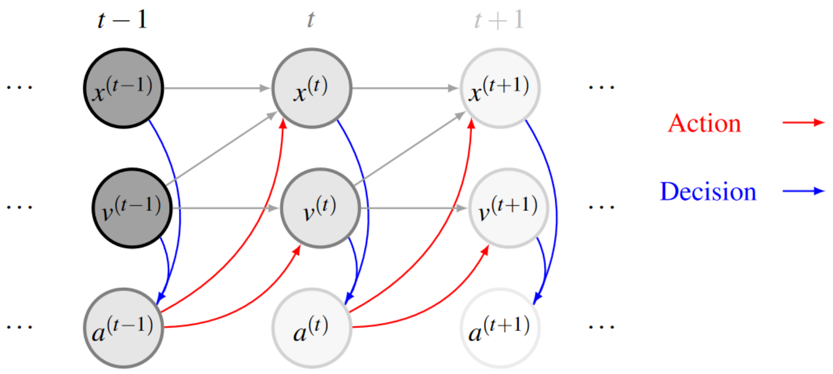
How to simulate $\delta(t+1)|\delta(t)$?

- **Dynamic IDM:**

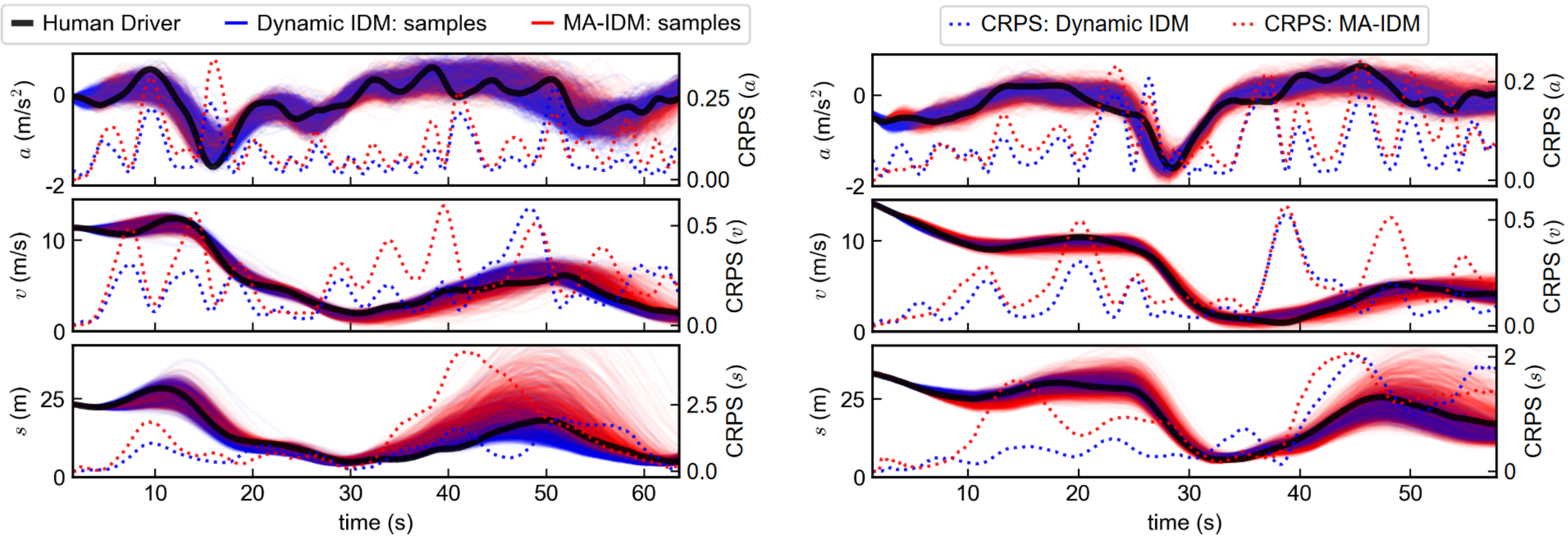
$$a_d^{(t)} = \mathrm{IDM}_d^{(t)} + \varepsilon_d^{(t)},$$
$$\varepsilon_d^{(t)} = \rho_{d,1}\varepsilon_d^{(t-1)} + \rho_{d,2}\varepsilon_d^{(t-2)} + \cdots + \rho_{d,p}\varepsilon_d^{(t-p)} + \eta_d^{(t)},$$
$$\eta^{(t)} \overset{i.i.d.}{\sim} \mathcal{N}(0, \sigma_\eta^2).$$

- **Stochastic simulation for step t_0:**

1. **generating the mean model by sampling a set of IDM parameters;**
2. **computing the serial correlation term according to the historical information;**
3. **sampling white noise randomly.**



Simulations – Stochastic Simulation (Dynamic IDM v.s. MA-IDM)



(a) Truck driver #211. (b) Car driver #273.

Brief results:

- **Action uncertainty is scenario specific**: When the leading vehicle is braking, all drivers have to decelerate; But when the leading vehicle accelerates, actions are more uncertain at their own will.
- Stochastic simulations can contain the ground truth curve in its envelope.
- Dynamic IDM has much lower variances than MA-IDM;

Simulations – Evaluation

Table 2: Evaluations of the short-term (5 s) simulations with different models. All values are amplified by ten times to keep an efficient form.

= real values ×10	RMSE(a)	RMSE(v)	RMSE(s)	CRPS(a)	CRPS(v)	CRPS(s)
MA-IDM	2.03 ± 0.48	3.00 ± 0.59	5.15 ± 0.86	1.11 ± 0.32	1.62 ± 0.39	3.14 ± 0.58
Bayesian IDM ($p = 0$)	3.19 ± 0.62	2.90 ± 0.83	6.00 ± 1.83	1.25 ± 0.33	1.92 ± 0.62	3.91 ± 1.29
Dynamic IDM ($p = 1$)	1.78 ± 0.54	2.83 ± 0.87	4.94 ± 1.46	1.26 ± 0.44	1.95 ± 0.67	3.07 ± 0.98
Dynamic IDM ($p = 2$)	1.74 ± 0.44	2.68 ± 0.66	4.78 ± 1.14	1.18 ± 0.36	1.77 ± 0.48	2.88 ± 0.76
Dynamic IDM ($p = 3$)	1.77 ± 0.46	2.77 ± 0.79	4.68 ± 1.26	1.10 ± 0.35	1.66 ± 0.55	2.51 ± 0.79
Dynamic IDM ($p = 4$)	1.76 ± 0.55	2.71 ± 0.85	4.43 ± 1.29	1.08 ± 0.42	1.64 ± 0.61	2.40 ± 0.83
Dynamic IDM ($p = 5$)	$\mathbf{1.66 \pm 0.38}$	2.65 ± 0.66	4.29 ± 1.01	$\mathbf{0.95 \pm 0.28}$	$\mathbf{1.49 \pm 0.45}$	$\mathbf{2.17 \pm 0.63}$
Dynamic IDM ($p = 6$)	1.76 ± 0.51	2.72 ± 0.81	4.41 ± 1.22	1.07 ± 0.39	1.60 ± 0.56	2.32 ± 0.76
Dynamic IDM ($p = 7$)	1.68 ± 0.39	2.65 ± 0.68	4.28 ± 1.09	1.00 ± 0.29	1.56 ± 0.46	2.24 ± 0.69
Dynamic IDM ($p = 8$)	1.68 ± 0.46	2.65 ± 0.74	4.27 ± 1.11	1.01 ± 0.35	1.55 ± 0.51	2.25 ± 0.71
Dynamic IDM ($p = 9$)	1.68 ± 0.47	$\mathbf{2.63 \pm 0.77}$	$\mathbf{4.23 \pm 1.17}$	1.01 ± 0.35	1.53 ± 0.54	2.23 ± 0.75
Dynamic IDM ($p = 10$)	1.72 ± 0.46	2.68 ± 0.76	4.27 ± 1.07	1.04 ± 0.34	1.56 ± 0.52	2.23 ± 0.65

Brief results:

- Dynamic IDM (AR+IDM) > MA-IDM (GP+IDM) >> Bayesian IDM > Traditional IDM

Lower variance

unbiased

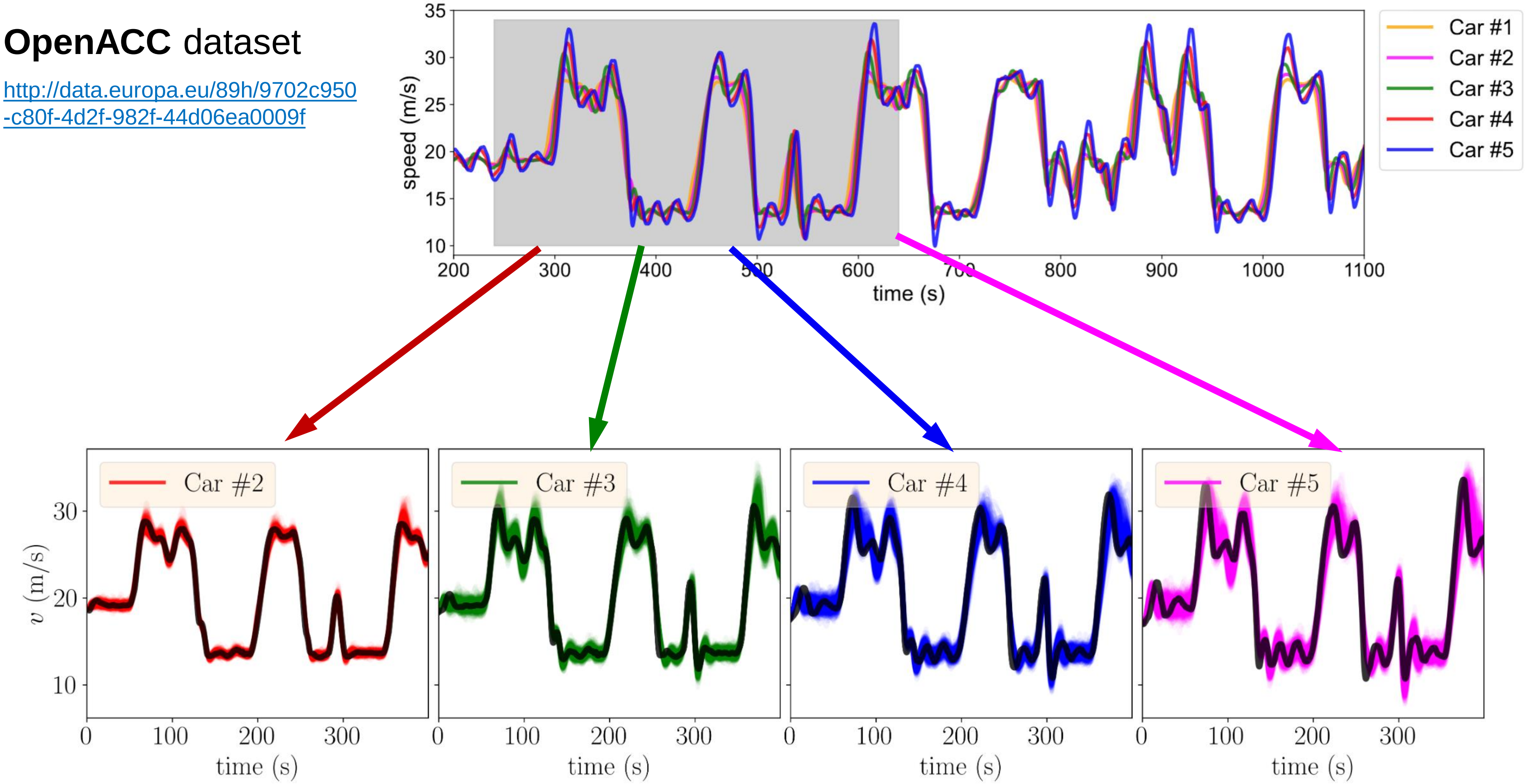
probabilistic

How about long-term multi-vehicle simulations?

Simulations – Multi-vehicle scenario: Platoon

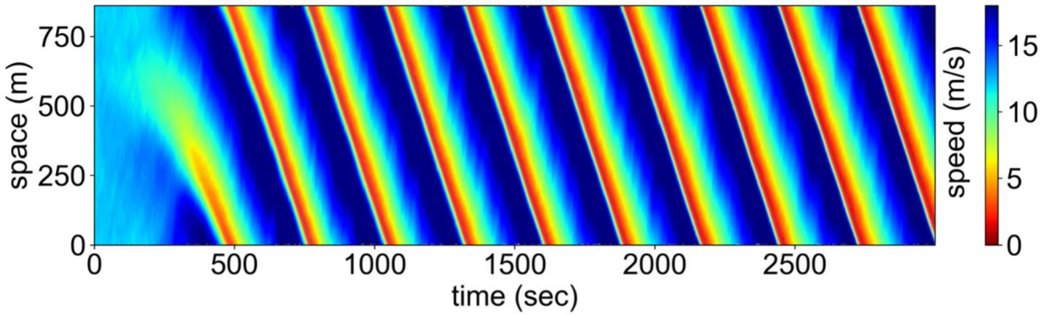
OpenACC dataset

http://data.europa.eu/89h/9702c950-c80f-4d2f-982f-44d06ea0009f

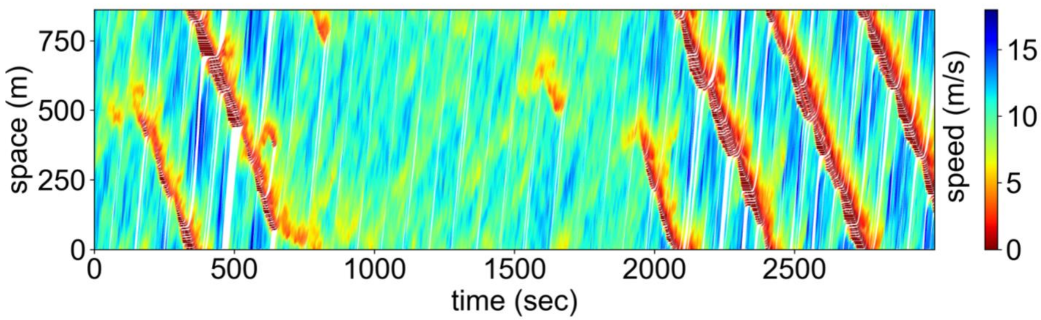


7 minute-simulations

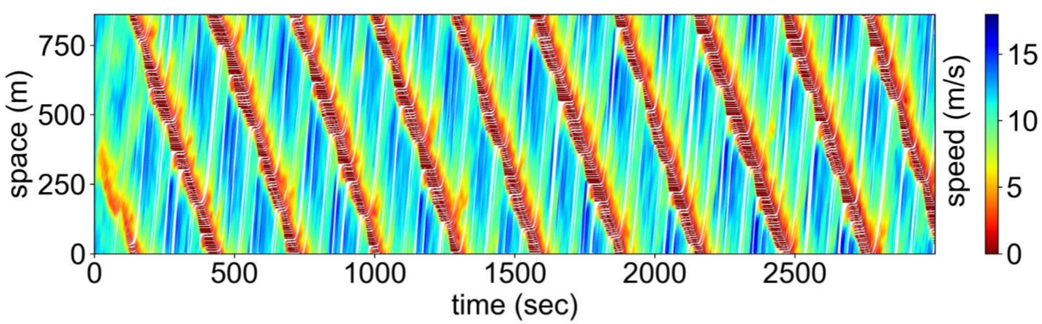
Simulations – Multi-vehicle scenario: Ring road



(a) Simulation with fixed IDM parameters and random white noise.



(b) Light traffic simulation with dynamic IDM ($p = 4$).



(c) Dense traffic simulation with dynamic IDM ($p = 4$).

Sugiyama experiment

General Overview

- **Real process:** $a(x,t) = a(x;\boldsymbol{\theta}) + \delta(t) + \epsilon, \; \epsilon \overset{i.i.d.}{\sim} \mathcal{N}(0, \sigma_\epsilon^2)$

IDM	MA-IDM (GP+IDM)	Dynamic IDM (AR+IDM)
$\boldsymbol{\theta}_{\text{IDM}}$ (5)	$\boldsymbol{\theta}_{\text{IDM}}, \ell, \sigma_k$ (7)	$\boldsymbol{\theta}_{\text{IDM}}, \boldsymbol{\rho}, \sigma_\eta$ (6+*d*)
i.i.d. white noise, bad uncertainty quantification	*Correlated error*, limited to kernel functions	*Correlated error*, good uncertainty quantification
$a^{(t)} = a_{\text{IDM}}^{(t)} + \epsilon_t$	$a^{(t)} = a_{\text{IDM}}^{(t)} + a_{\text{GP}}^{(t)}$	$a^{(t)} = a_{\text{IDM}}^{(t)} + \eta^{(t)} + \sum_p \rho_p (\hat{a}^{(t-p)} - a_{\text{IDM}}^{(t-p)})$

Discussion and takeaway

✓ Generate diverse types of drivers. [Bayesian calibration/Hierarchical structure]

✓ Produce good uncertainty for each driver. [GP/AR]

✓ Simulate human-like car-following behaviors. [Stochastic Simulation]

➢ **Importance of probabilistic simulation!**

➢ positive correlations (0~5 sec) & negative correlations (5~10 sec)

→ at least 10 sec historical information as input.

➢ Provide enough information to calibrate car-following models.

➢ IDM is very powerful.

References

- Treiber, M., Hennecke, A., & Helbing, D. (2000). Congested traffic states in empirical observations and microscopic simulations. Physical Review E, 62(2), 1805.
- Punzo, V., Zheng, Z., & Montanino, M. (2021). About calibration of car-following dynamics of automated and human-driven vehicles: Methodology, guidelines and codes. Transportation Research Part C: Emerging Technologies, 128, 103165.
- Krajewski, R., Bock, J., Kloeker, L., & Eckstein, L. (2018). The highd dataset: A drone dataset of naturalistic vehicle trajectories on german highways for validation of highly automated driving systems. In 2018 21st International Conference on Intelligent Transportation Systems (ITSC) (pp. 2118-2125). IEEE.
- Anesiadou, A., Makridis, M., Ciuffo, B., & Mattas, K. (2020): Open ACC Database. European Commission, Joint Research Centre (JRC) [Dataset] PID: http://data.europa.eu/89h/9702c950-c80f-4d2f-982f-44d06ea0009f
- Treiber, M., Kesting, A., & Helbing, D. (2006). Delays, inaccuracies and anticipation in microscopic traffic models. Physica A: Statistical Mechanics and its Applications, 360(1), 71-88.

Read More

- **Paper:**

Zhang, C., & Sun, L. (2024). Bayesian calibration of the intelligent driver model. *IEEE Transactions on Intelligent Transportation Systems*.

Zhang, C., Wang, W., & Sun, L. (2024). Calibrating car-following models via Bayesian dynamic regression. Transportation Research Part C: Emerging Technologies, 104719.

- **Code:**

https://github.com/Chengyuan-Zhang/IDM_Bayesian_Calibration

Thanks!
Questions?

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